The SIML Estimation of Integrated Volatility, Covariances and Hedging Coefficients under Micro-market Noise, Round-off Errors and Random Sampling [a]

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[a] This talk is based on unpublished papers with Seisho Sato and Hiroumi Misaki (University of Tokyo) which are available at http://www.e.u-tokyo.ac.jp/cirje/research/dp.

Outline of Presentation

1. Introduction
2. Problems of Micro-market adjustments, Round-off errors and Random Sampling
3. SIML (Separating Information Maximum Likelihood) estimation of Integrated Volatility, Covariance and Hedging Coefficients
4. Asymptotic Properties and Robustness
5. Simulations
6. Concluding Remarks

1 Motivations of Study

1. Recently a considerable interest has been paid on the estimation problem of the integrated volatility by using (ultra) high-frequency financial data. Several statistical methods have been developed by Anderson, T.G., Bollerslev, T. Diebold, F.K. and Labys, P. (2000 JASA), Gloter and Jacod (2001), Ait-Sahalia, Y., P. Mykland and L. Zhang (2005), Zhang, L., P. Mykland and Ait-Sahalia (2005), Hayashi and Yoshida (2005), Barndorff-Nielsen, O., P. Hansen, A. Lunde and N. Shepard (2008, 2011), and Malliavin and Mancino (2009).

2. Our aim is to develop a simple (non-parametric) estimation method for practical applications with micro-market noise. We have proposed to use the SIML (Separating Information Maximum Likelihood) method by Kunitomo and Sato (2008, 2011).

3. We investigate the robustness property of the SIML estimation when we have the micro-market adjustment mechanism, the round-off errors and *Random Sampling* in the process forming the observed prices. The micro-market models including the price adjustments mechanisms have been discussed in the *micro-market literature* in financial economics. We consider the nonlinear price adjustment models while we regard a continuous martingale as the hidden intrinsic value of underlying security including the round-off error models when the high frequency data are randomly sampled.

4. The SIML estimation of the integrated variance, covariance and the hedging coefficients are asymptotically robust in these situations; that is, they are consistent and asymptotically normal (in the meaningful sense) as the sample size increases under a reasonable set of assumptions. The asymptotic robustness of the SIML method for the underlying continuous stochastic process with micro-market noise in the multivariate non-Gaussian cases.

Problem : Estimation of Volatility,Covariances and Hedging Coefficients

- Historical Volatility and Historical Covariances
- Non-linear time series models (ARCH, SV, GARCH etc.)
- By using high-frequency data, the estimation of the integrated volatility and covariances

  daily data －＞ 1 minitue data, 1 second, Tick data etc.

  ＊possibility of finer estimation of Volatility ?

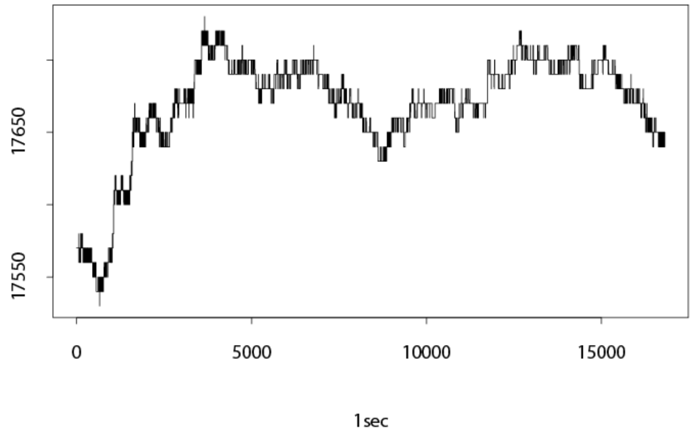
・ The Role of Micro-market noises : It can be interpreted as statistical measurement errors

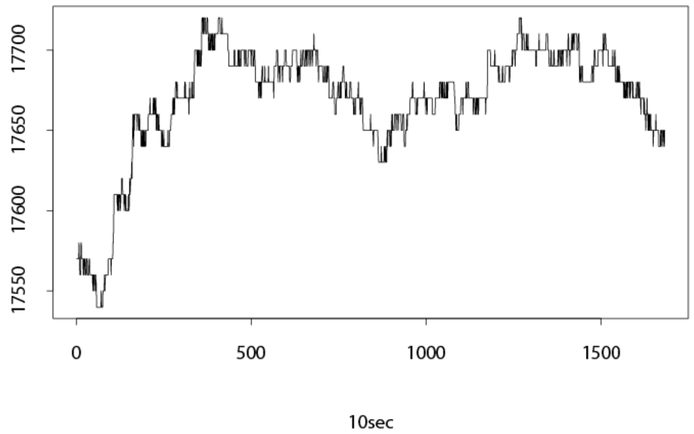
・ Several alternative methods developed

High Frequency Data of Nikkei-225 Futures

- Traded at Osaka Securites Exchange(OSE)
- Very active
  - Daily average volume 136,802 units (2008)
- Intra-day volatility movements
- Large tick size
  - 10 yen (Spot index : 0.01 yen)

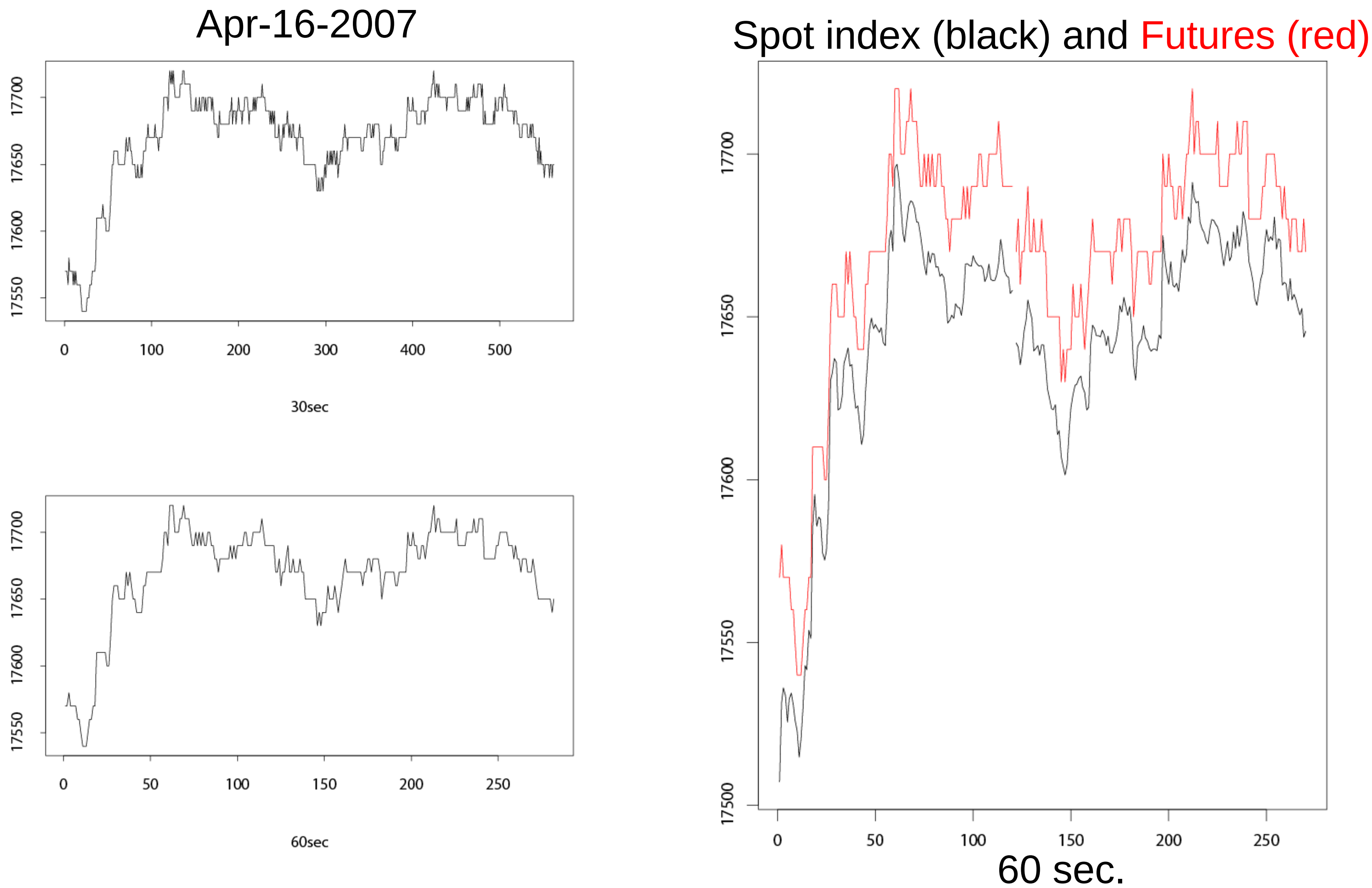
Apr-16-2007





*Original Futures data are provided by Osaka Securities Exchange.

High Frequency Data



2 Micro-market adjustments, the Round-off error models and Random Sampling

A General Formulation

We consider $\mathbf{y}_{sf} = (y_s(t_i^s), y_f(t_j^f))'$ such that

$$y_s(t_i^s) = h_s\left(X(t), y_s(t_{i-1}^s), u_s(t_i^s)\ ,\ 0 \le t \le t_i^s\right)\ (i = 1, \cdots, n_s^*)\ ,$$

$$y_f(t_j^f) = h_f\left(X(t), y_f(t_{j-1}^f), u_f(t_j^f)\ ,\ 0 \le t \le t_j^f\right)\ (j = 1, \cdots, n_f^*)$$

and

$$X_t = X_0 + \int_0^t C_x(s) dB_s\ (0 \le t \le 1)\ ,$$

where the (unobservable) continuous martingale process $X(t)$ generated by Brownian motions and $u(t_i^s)$ and $u(t_j^f)$ are the micro-market noises.
For the simplicity, we set $p = q = 2$ (the dimensions of observed variables and Brownian Motions, respectively) and assume that
$\mathcal{E}(u_s(t_i^s)) = 0, \mathcal{E}(u_f(t_j^f)) = 0$; $\mathcal{E}(u_s(t_i^s)^2) = \sigma_{ss}^{(u)}$, $\mathcal{E}(u_f(t_j^f)^2) = \sigma_{ff}^{(u)}$,

$\mathcal{E}(u_s(t_i^s)u_f(t_j^f)) = \sigma_{sf}^{(u)}\delta(t_i^s, t_j^f)$; $0 = t_0^s \leq t_1^s \leq \cdots \leq t_{n_s^*}^s$,
$0 = t_0^f \leq t_1^f \leq \cdots \leq t_{n_f^*}^f$, and $h_s(\cdot)$ and $h_f(\cdot)$ are measurable functions.

We want to estimate
(i) the integrated volatilities $\int_0^1 \sigma_{ss}^{(x)}(s)ds$ (and $\int_0^1 \sigma_{ff}^{(x)}(s)ds$),
(ii) the integrated covariance $\int_0^1 \sigma_{sf}^{(x)}(s)ds$
and
(iii) the hedging coefficient $H = \int_0^1 \sigma_{sf}^{(x)}(s)ds / \int_0^1 \sigma_{ff}^{(x)}(s)ds$.

Assumption 2.1 : There exist positive constants $c_{(a)}$ $(a = s, f)$ such that

$$\max_i t_i^{(a)} \longrightarrow 1 \ , \ \frac{n_{(a)}^*}{n} \xrightarrow{p} c_{(a)}$$

and

$$\mathcal{E}\left[|t_i^{(a)} - t_{i-1}^{(a)}| \right] = O(n^{-1})$$

as $n \to \infty$, where $a = s$ or $a = f$ and $n_{(a)}^*$ are the (random) sample sizes. (n is an index of sample size and we set $c_{(a)} = 1$ without loss of generality.)

Assumption 2.2 : The stochastic process $X(t)$ $(0 \le t \le 1)$ is independent of the random sequences t_i^s and t_j^f $(j \ge 1)$.

Example 1 (Equi-distance Sampling) : $t_i^{(a)} - t_{i-1}^{(a)} = 1/n$ ($a = s$ or $a = f$) and $i = 1, \cdots, n$.

Example 2 (Poisson Random Sampling) : $t_i^{(a)}$ ($a = s$ or $a = f$) follows the Poisson Process with λ_n $(= c_{(a)} n)$.

Example 3 (EACD(1,1)) (Engle=Russel (2008), Autoregressive Conditional Duration Models) : Let $\tau_i^{(a)} = t_i^{(a)} - t_{i-1}^{(a)}$ and $\tau_i^{(a)} = \psi_i^{(a)} \epsilon_i^{(a)}$ such that

$$\psi_i^{(a)} = \omega^{(a)} + \alpha^{(a)} \tau_i^{(a)} + \beta^{(a)} \psi_{i-1}^{(a)}$$

and $\epsilon_i^{(a)}$ are the sequence of i.i.d. exponential random variables with $\alpha^{(a)} > 0, \beta^{(a)} > 0$ and $\omega^{(a)} > 0$.

(i) **Basic Additive Model**

When $p = q = 1$, the basic additive model is represented by the observed log-price $y(t_i^n)$ as

$$y(t_i^n) = X(t_i^n) + u(t_i^n) \quad ,$$

where the continuous martingale is given by

$$X_t = X_0 + \int_0^t c_x(s) dB_s \ (0 \le t \le 1) \ ,$$

$\Sigma_x(s) = \sigma_x^2(s) = c_x^2(s)$ (the instantaneous volatility) and $u(t_i^n)$ is the micro-market noise.

We want to estimate the integrated volatility $\Sigma_x = \sigma_x^2 = \int_0^1 c_x^2(s) ds$.

There are several important cases of the present formulation for modeling the financial markets with high frequency financial data.

(ii) **A Micro-market price Adjustment model**

We set $y_i^{(a)} = P(t_i^{(a)})$ and $x_i^{(a)} = X(t_i^{(a)})$. We consider the (linear) micro-market price adjustment model

$$P^{(a)}(t_i^{(a)}) - P^{(a)}(t_{i-1}^{(a)}) = g^{(a)} \left[X^{(a)}(t_i^{(a)}) - P^{(a)}(t_{i-1}^{(a)}) \right] + u^{(a)}(t_i^{(a)}) \ ,$$

where $X(t)$ (the intrinsic vector of securities at t) and $P^{(a)}(t_i^{(a)})$ are measured in logarithms, the adjustment (constant) coefficient $g^{(a)}$ ($0 < g^{(a)} < 2$), and $u^{(a)}(t_i^{(a)})$ are i.i.d. sequence of noises with $\mathcal{E}[u^{(a)}(t_i^{(a)})] = 0$ and $\mathcal{E}[u^{(a)}(t_i^{(a)})^2] = \sigma_{aa}^{(u)}$.

(iii) **The Round-off-error model**

We assume that

$$P^{(a)}(t_i^{(a)}) - P^{(a)}(t_{i-1}^{(a)}) = g_\eta^{(a)} \left[X^{(a)}(t_i^{(a)}) - P^{(a)}(t_{i-1}^{(a)}) + u^{(a)}(t_i^{(a)}) \right] \ ,$$

where $u^{(a)}(t_i^{(a)})$ is an i.i.d. sequence of noise with $\mathcal{E}[u^{(a)}(t_i^{(a)})] = 0$,$\mathcal{E}[u(t_i^{(a)})^2] = \sigma_{aa}^{(u)}$ and the nonlinear function

$$g_\eta^{(a)}(x) = \eta \left[\frac{x}{\eta} \right] \ ,$$

where $g_\eta(y)$ is the integer part of y and $[y]$ is the largest integer being less than y and η is a small positive constant.

This model corresponds to the micro-market model with the restriction of the minimum price change and η is the parameter of minimum price change. We set $y_i^{(a)} = P(t_i^{(a)})$ and $x_i^{(a)} = X(t_i^{(a)})$.

(iv) **Nonlinear Micro-market price Adjustment models**

We take a non-linear version with

$$g(x) = g_1 x \mathrm{I}(x \geq 0) + g_2 x \mathrm{I}(x < 0) \ ,$$

where $g_i \ (i = 1, 2)$ are some constants and $\mathrm{I}(\cdot)$ is the indicator function. (This has been called the SSAR (simultaneous switching autoregressive) model, whic have been investigated by Sato and Kunitomo (1996) and Kunitomo and Sato (1999).) A set of sufficient conditions for the geometric ergodicity of the price process is given by $g_1 > 0 \ , \ g_2 > 0 \ , \ (1 - g_1)(1 - g_2) < 1 \ .$

More generally, we consider the model

$$P(t_i^{(a)}) - P(t_{i-1}^{(a)}) = g\left[X(t_i^{(a)}) - P(t_{i-1}^{(a)})\right] + u(t_{i-1}^{(a)}) \ ,$$

where $u(t_i^{(a)})$ is an i.i.d. sequence of noise with $\mathcal{E}[u(t_i^{(a)})] = 0$ and $\mathcal{E}[u(t_i^{(a)})^2] = \sigma_{aa}^u$. We set $y_i^{(a)} = P(t_i^{(a)})$ and $x_i^{(a)} = X(t_i^{(a)})$.

3 SIML: Separating Information Maximum Likelihood) estimation

Let y_{ij} be the $i-$th observation of the $j-$th (log-) price at t_i^n in the equi-distance case, that is, for $j = 1, \cdots, p; i = 1, \cdots, n$ $0 = t_0^n \leq t_1^n \leq \cdots \leq t_n^n = 1$. In the general case, this paper uses the refreshing-time method.

We set $\mathbf{y}_i = (y_{i1}, \cdots, y_{ip})^{'}$ be a $p \times 1$ vector and $\mathbf{Y}_n = (\mathbf{y}_i^{'})$ be an $n \times p$ matrix of observations. The underlying continuous process $\mathbf{x}_i$ at t_i^n $(i = 1, \cdots, n)$ is not necessarily the same as the observed prices and let $\mathbf{v}_i^{'} = (v_{i1}, \cdots, v_{ip})$ be the vector of the additive micro-market noise at t_i^n, which is independent of $\mathbf{x}_i$. Then we have

$$\mathbf{y}_i = \mathbf{x}_i + \mathbf{v}_i$$

where $\mathbf{v}_i$ are a sequence of independent random variables with $\mathcal{E}(\mathbf{v}_i) = \mathbf{0}$ and $\mathcal{E}(\mathbf{v}_i \mathbf{v}_i^{'}) = \mathbf{\Sigma}_v$. We sometimes have focused on the equi-distance (one-dimension) case with $h_n = t_i^n - t_{i-1}^n = 1/n$ $(i = 1, \cdots, n)$ and

Standard Framework of Analysis

Let y_{ij} be the $i-$th observation of the $j-$th (log-) price at t_i^n $(j = 1, \cdots, p; 0 = t_0^n \le t_1^n \le \cdots \le t_n^n = 1)$, $\mathbf{y}_i = (y_{i1}, \cdots, y_{ip})$ be a $p \times 1$ vector and $\mathbf{Y}_n = (\mathbf{y}_i^{'})$ be an $n \times p$ matrix of observations.
The underlying continuous process $\mathbf{x}_i$ is not necessarily the same as the observed prices and $\mathbf{v}_i^{'} = (v_{i1}, \cdots, v_{ip})$ is the vector of the micro-market noise :

$$\begin{aligned} \mathbf{y}_i &= \mathbf{x}_i + \mathbf{v}_i \ , \\ \mathbf{x}_t &= \mathbf{x}_0 + \int_0^t \mathbf{\Sigma}_x^{1/2}(s) d\mathbf{B}_s \ \ (0 \le t \le 1), \end{aligned}$$

where $\mathcal{E}(\mathbf{v}_i) = \mathbf{0}$, $\mathcal{E}(\mathbf{v}_i \mathbf{v}_i^{'}) = \mathbf{\Sigma}_v$, $\mathbf{B}_s$ is a $p \times 1$ vector of the standard Brownian motions and $\mathbf{\Sigma}_x(s) = \mathbf{\Sigma}_x^{1/2}(s) \mathbf{\Sigma}_x^{1/2}(s)^{'}$.

Discrete model:

$$x_i = x_{i-1} + e_i$$

$$y_i = x_i + v_i$$

SIML (Separating Information Maximum Likelihood) Estimation

proposed by Kunitomo and Sato (2008, 2011)

We transform $\mathbf{Y}_n$ to $\mathbf{Z}_n$ $(= (\mathbf{z}_k^{'}))$ by

$$\mathbf{Z}_n = h_n^{-1/2}\mathbf{P}_n^{'}\mathbf{C}_n^{-1}\left(\mathbf{Y}_n - \bar{\mathbf{Y}}_0\right)$$

where

$$\bar{\mathbf{Y}}_0 = \mathbf{1}_n \otimes \mathbf{y}_0^{'}\ .$$

$$\mathbf{P}_n = (p_{jk})\ ,\ p_{jk} = \sqrt{\frac{2}{n+\frac{1}{2}}}\cos\left[\pi(\frac{2k-1}{2n+1})(j-\frac{1}{2})\right]$$

$$\mathbf{C}_n^{-1} = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 \\ -1 & 1 & 0 & \cdots & 0 \\ 0 & -1 & 1 & 0 & \cdots \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{pmatrix}$$

P_n: rotation matrix

Then **the likelihood function under the Gaussian noises** is given by

$$L_n^*(\boldsymbol{\theta}) = \left(\frac{1}{\sqrt{2\pi}}\right)^{np} \prod_{k=1}^{n} |a_{kn}\boldsymbol{\Sigma}_v + \boldsymbol{\Sigma}_x|^{-1/2} e^{\left\{-\frac{1}{2}\mathbf{z}_k^{'}\left(a_{kn}\boldsymbol{\Sigma}_v + \boldsymbol{\Sigma}_x\right)^{-1}\mathbf{z}_k\right\}} ,$$

where

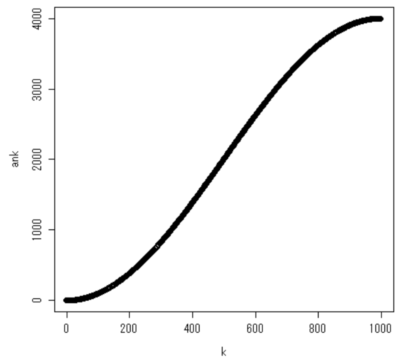
$$a_{k,n} = 4n\sin^2\left[\frac{\pi}{2}\left(\frac{2k-1}{2n+1}\right)\right] \quad (k = 1, \cdots, n) .$$

Hence the maximum likelihood (ML) estimator can be defined as the solution of minimizing

$$L_n(\boldsymbol{\theta}) = -\sum_{k=1}^{n} \log|a_{k,n}\boldsymbol{\Sigma}_v + \boldsymbol{\Sigma}_x|^{-1/2} + \frac{1}{2}\sum_{k=1}^{n} \mathbf{z}_k^{'}[a_{k,n}\boldsymbol{\Sigma}_v + \boldsymbol{\Sigma}_x]^{-1}\mathbf{z}_k .$$

For small *k*,

$$a_{n,k} = 4n\sin^2\left[\frac{\pi}{2}\left(\frac{2k-1}{2n+1}\right)\right] \cong 0$$



The SIML Estimation : We approximate $2 \times L_n(\boldsymbol{\theta})$ by

$$L_{1n}(\boldsymbol{\theta}) = m \log |\boldsymbol{\Sigma}_x| + \sum_{k=1}^{m} \mathbf{z}_k^{'} \boldsymbol{\Sigma}_x^{-1} \mathbf{z}_k \ .$$

Then the SIML estimator of $\hat{\boldsymbol{\Sigma}}_x$ is defined by

$$\hat{\boldsymbol{\Sigma}}_x = \frac{1}{m} \sum_{k=1}^{m} \mathbf{z}_k \mathbf{z}_k^{'} \ .$$

We also approximate $2 \times L_n(\boldsymbol{\theta})$ by

$$L_{2n}(\boldsymbol{\theta}) = \sum_{k=n+1-l}^{n} \log |a_{k,n} \boldsymbol{\Sigma}_v| + \sum_{k=n+1-l}^{n} \mathbf{z}_k^{'} [a_{k,n} \boldsymbol{\Sigma}_v]^{-1} \mathbf{z}_k \ .$$

Then the SIML estimator of $\hat{\boldsymbol{\Sigma}}_v$ is defined by

$$\hat{\boldsymbol{\Sigma}}_v = \frac{1}{l} \sum_{k=n+1-l}^{n} a_{k,n}^{-1} \mathbf{z}_k \mathbf{z}_k^{'} \ .$$

$$\underbrace{z_1, z_2, \cdots, z_m}_{\downarrow \atop \hat{\Sigma}_x}, \cdots \cdots, \underbrace{z_{n-l+1}, z_{n-l+2}, \cdots, z_n}_{\downarrow \atop \hat{\Sigma}_v}$$

Improving SIML

It is possible to improve the bias term and the convergence rate.

(i) Bias Correction

Define the modified SIML (MSIML) is given by

$$\hat{\Sigma}_{x,MSIML} = \hat{\Sigma}_{x,SIML} - [\frac{1}{m_n}\sum_{k=1}^{m_n} a_{kn}]\hat{\Sigma}_{v,SIML}$$

Then we can imporve the Asymptotic BIAS in some extreme cases. (In practical cases we often do not need this correction.)

(ii) Optimal Convergence Rate

If we make Bias Correction, the MSIML can attain the Optimal Convergence Rate as well as Asymptotic Robustness by choosing the parameters m and l or $\alpha\ and\ \beta$.

(iii) There have been some recent developments and the related problems have been under further investigation. (It will be reported in a systematic way.)

4 Asymptotic Properties of the SIML and Asymptotic Robustness

4.1 A summary of Asymptotic Properties

We summarize the asymptotic properties of the SIML estimator when the sample size n is large. Kunitomo and Sato [2008, 2011] have investigated the problem and have shown that the SIML estimator is consistent and it has the asymptotic normality under a set regularity conditions when $p = q = 1$.
As $n \longrightarrow \infty$

$$\hat{\sigma}_x^2 - \sigma_x^2 \xrightarrow{p} 0$$

with $m_n = n^\alpha$ $(0 < \alpha < 1/2)$ and

$$\sqrt{m_n}\left[\hat{\sigma}_x^2 - \sigma_x^2\right] \xrightarrow{d} N\left(0, 2[\sigma_x^2]^2\right)$$

with $m_n^5/n^2 \to 0$.

Although the SIML estimation was introduced under the Gaussian process and the standard model, it has reasonable finite sample properties as well as asymptotic properties under some volatility models and the non-Gaussian processes with

$$\mathcal{E}\left[(x_i - x_{i-1})^2 | \mathcal{F}_{n,i-1}\right] = \int_{t_{i-1}}^{t_i} C_x^2(s) ds \ .$$

As $n \longrightarrow \infty$, under a set of regularity conditions, the asymptotic distribution of the SIML estimator of the integrated variance can be summarized as

$$\sqrt{m_n}\left[\hat{\sigma}_{xx} - \sigma_{xx}\right] \xrightarrow{d} N\left[0, V_{xx}\right] \ ,$$

provided that we have the convergence of the asymptotic variance

$$V_{xx} = 2\left[\int_0^1 C_x^4(s) ds\right]$$

and it is a positive constant when $m_n^5/n^2 \rightarrow 0$ (as $n \rightarrow \infty$).

Remark : When V_{xx} is a random variable, the convergence is in the sense of *stable convergence*.

When $p = q = 2$, as $n \longrightarrow \infty$, under a set of regularity conditions, the asymptotic distribution of the SIML estimator of integrated covariance can be summarized as

$$\sqrt{m_n}\left[\hat{\sigma}_{sf} - \sigma_{sf}\right] \xrightarrow{d} N\left[0, V_{sf}\right] \ ,$$

provided that we have the convergence of the asymptotic variance

$$V_{sf} = \int_0^1 \left[\sigma_{ss}^{(x)}(s)\sigma_{ff}^{(x)}(s) + \sigma_{sf}^2(s)\right] ds \ .$$

4.2 Asymptotic Robustness under Micro-market adjustments and the Round-off error models

(i) **A Micro-market price Adjustment model**

Theorem 3-3 (Sato-Kunitomo) : Assume $0 < g < 2$ and Define the SIML estimator of the realized volatility of $X(t)$ with $m_n = n^{\alpha}$ $(0 < \alpha < 0.4)$. Then the asymptotic distribution of $\sqrt{m_n}\left[\hat{\sigma}_x^2 - \sigma_x^2\right]$ is asymptotically $(m_n, n \to \infty)$ equivalent to the limiting distributions under the standard additive (i.e. the signal-plus-noise) models.

(ii) **The Round-off-error model**

Theorem 3.4 (Sato-Kunitomo) : Set $\eta = \eta_n$ depending on n with $\eta_n \sqrt{n} = O(1)$. Define the SIML estimator of the realized volatility of $X(t)$ with $m_n = n^{\alpha}$ $(0 < \alpha < 0.4)$. The limiting random variable of the normalized estimator $\sqrt{m_n}\left[\hat{\sigma}_x^2 - \sigma_x^2\right]$ is asymptotically $(m_n, n \to \infty)$ equivalent to the limiting distributions in the standard models.

(iii) **Nonlinear Micro-market price Adjustment models**

Theorem 3-5 (Sato-Kunitomo) : For the non-linear time series process $V(t_i^n)$ we assume that there exist functions $\rho_1(\ \cdot\ )$ and $\rho_2(\ \cdot,\cdot\ )$ such that $\mathrm{Cov}[V(t_i^n), V(t_j^n)] = c_1\rho_1(|i-j|)$, where c_1 is a (positive) constant and $\sum_{s=0}^{\infty} \rho_1(s) < \infty$ and
$\mathrm{Cov}\left[V(t_i^n)V(t_{i'}^n), V(t_j^n)V(t_{j'}^n))\right] = c_2\rho_2(|i-i^{'}|, |j-j^{'}|)$, where c_2 is a (positive) constant and $\sum_{s,s^{'}=0}^{\infty} \rho_2(s,s^{'}) < \infty$.
Define the SIML estimator of the realized volatility of $P(t_i^n)$ with $m_n = n^{\alpha}$ $(0 < \alpha < 0.4)$. Then the asymptotic distribution of $\sqrt{m_n}\left[\hat{\sigma}_x^2 - \sigma_x^2\right]$ is asymptotically (as $m_n, n \to \infty$) equivalent to the limiting distributions under the standard models.

5 Simulations

5-1 Simulations in Sato-Kunitomo (2011)

In our simulation the the volatility function is given by

$$\sigma_x^2(s) = \sigma(0)^2 \left[a_0 + a_1 s + a_2 s^2 \right] ,$$

where a_i $(i = 0, 1, 2)$ are constants and we have some restrictions such that $\sigma_x(s)^2 > 0$ for $s \in [0, 1]$. It is a typical time varying (but deterministic) case and the realized variance is given by

$$\sigma_x^2 = \int_0^1 \sigma_x(s)^2 ds = \sigma_x(0)^2 \left[a_0 + \frac{a_1}{2} + \frac{a_2}{3} \right] .$$

We have examined several models

Model 1 $\quad h_1(x,y,u) = y + g(x-y) + u \ (g : \text{a const})$,

Model 2 $\quad h_2(x,y,u) = y + g_\eta(x-y+u) \ \ (g_\eta(\cdot) \text{ is } (3.8))$,

Model 3 $\quad h_3(x,y,u) = y + g_\eta(x-y) + u \ \ (g_\eta(\cdot) \text{ is } (3.8))$,

Model 4 $$h_4(x,y,u) = y + u + \begin{cases} g_1(x-y) & \text{if } y \geq 0 \ (g_1 : \text{a const}) \\ g_2(x-y) & \text{if } y < 0 \ (g_2 : \text{a const}) \end{cases} ,$$

Model 5 $\quad h_5(x,y,u) = y + \left[g_1 + g_2 \exp(-\gamma|x-y|^2)\right](x-y) \ (g_1, g_2 : \text{const})$,

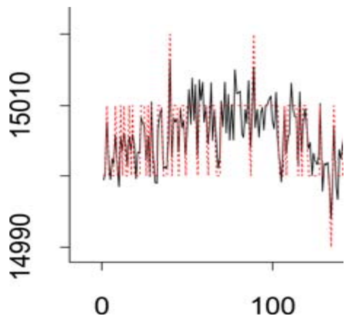
Model 6 $\quad h_6(x,y,u) = y + g_1 \sin\left(g_2(x-y)\right) \ \ (g_1, g_2 : \text{const})$,

Model 7 $\quad h_7(x,y,u) = y + h_2 \ o \ h_4 \ o \ h_1(x,y,u)$,

respectively.

Table A-3 : Round Error Model ($\alpha = 0.4$)

n=300	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol
True	5.00E-05	5.00E-07		5.00E-05	5.00E-08		5.00E-05	0.00E+00	
Mean	5.14E-05	5.82E-07	3.73E-04	5.09E-05	1.32E-07	1.02E-04	4.99E-05	8.24E-08	7.23E-05
SD	2.41E-05	8.39E-08	3.56E-05	2.39E-05	1.91E-08	8.85E-06	2.40E-05	1.15E-08	5.87E-06
MSE	5.81E-10	1.38E-14		5.70E-10	7.17E-15		5.74E-10	6.92E-15	
AVAR	5.11E-10	5.22E-15		5.11E-10	5.22E-17		5.11E-10	0.00E+00	
n=5000	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol
True	5.00E-05	5.00E-07		5.00E-05	5.00E-08		5.00E-05	0.00E+00	
Mean	5.13E-05	5.39E-07	5.42E-03	5.04E-05	8.94E-08	9.19E-04	5.01E-05	2.04E-08	2.66E-04
SD	1.35E-05	2.50E-08	1.33E-04	1.29E-05	4.22E-09	2.24E-05	1.31E-05	1.68E-09	1.43E-05
MSE	1.84E-10	2.16E-15		1.67E-10	1.57E-15		1.71E-10	4.20E-16	
AVAR	1.66E-10	5.49E-16		1.66E-10	5.49E-18		1.66E-10	0.00E+00	
n=20000	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol
True	5.00E-05	5.00E-07		5.00E-05	5.00E-08		5.00E-05	0.00E+00	
Mean	5.11E-05	5.38E-07	2.15E-02	5.03E-05	8.74E-08	3.52E-03	5.02E-05	1.02E-08	5.32E-04
SD	1.00E-05	1.46E-08	2.61E-04	9.70E-06	2.46E-09	5.05E-05	9.86E-06	7.50E-10	2.93E-05
MSE	1.01E-10	1.64E-15		9.42E-11	1.41E-15		9.72E-11	1.04E-16	
AVAR	9.52E-11	1.81E-16		9.52E-11	1.81E-18		9.52E-11	0.00E+00	



Data generating process: $y_t = \log(y'_t)$

$$y'_t = 10 \times floor(exp(y''_t)/10 + 0.5)$$

$$y''_t = x_t + v_t$$

$$x_t = x_{t-1} + u_t, x_0 = \log(15000)$$

$$u_t \sim i.i.d.N(0, \sigma_x^2/n), v_t \sim i.i.d.N(0, \sigma_v^2)$$

* $floor(x)$ is a function whose value is the largest integer less than or equal to x.

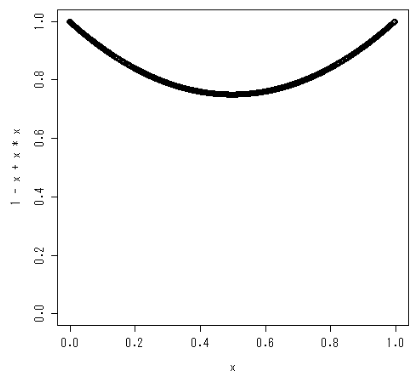
Table A-5 : Estimation of Realized Volatility (U-shaped volatility, $\alpha = 0.4$)

n=300	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol
True	1.67E-04	2.00E-06		1.67E-04	2.00E-07		1.67E-04	2.00E-09	
Mean	1.71E-04	2.15E-06	1.37E-03	1.68E-04	3.53E-07	2.88E-04	1.68E-04	1.53E-07	1.68E-04
SD	8.17E-05	3.18E-07	1.33E-04	7.91E-05	5.14E-08	2.45E-05	7.96E-05	2.26E-08	1.39E-05
MSE	6.68E-09	1.23E-13		6.25E-09	2.59E-14		6.33E-09	2.34E-14	
AVAR	5.67E-09	8.34E-14		5.67E-09	8.34E-16		5.67E-09	8.34E-20	
n=5000	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol
True	1.67E-04	2.00E-06		1.67E-04	2.00E-07		1.67E-04	2.00E-09	
Mean	1.72E-04	2.01E-06	2.02E-02	1.67E-04	2.09E-07	2.17E-03	1.67E-04	1.06E-08	1.87E-04
SD	4.52E-05	9.34E-08	4.96E-04	4.27E-05	9.87E-09	5.22E-05	4.30E-05	4.94E-10	3.76E-06
MSE	2.07E-09	8.76E-15		1.82E-09	1.75E-16		1.85E-09	7.38E-17	
AVAR	1.84E-09	8.79E-15		1.84E-09	8.79E-17		1.84E-09	8.79E-21	
n=20000	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol	Σ_x	Σ_v	H-vol
True	1.67E-04	2.00E-06		1.67E-04	2.00E-07		1.67E-04	2.00E-09	
Mean	1.71E-04	2.00E-06	8.02E-02	1.68E-04	2.02E-07	8.17E-03	1.67E-04	4.12E-09	2.47E-04
SD	3.47E-05	5.43E-08	9.68E-04	3.32E-05	5.39E-09	9.86E-05	3.31E-05	1.12E-10	2.54E-06
MSE	1.22E-09	2.95E-15		1.11E-09	3.34E-17		1.09E-09	4.49E-18	
AVAR	1.06E-09	2.90E-15		1.06E-09	2.90E-17		1.06E-09	2.90E-21	

Data generating process:

$$y_t = x_t + v_t,\ x_t = x_{t-1} + u_t$$

$$u_t \sim i.i.d.N(0, (1 - s + s^2)\sigma_x^2/n), v_t \sim i.i.d.N(0, \sigma_v^2), s = t/n$$



Simulations for integrated covariance

Table A-6 : Correlation ($\alpha = 0.4$,$corv = 0$)

n=300	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx
True	5.00E-05	5.00E-07	9.00E-01	5.00E-05	5.00E-08	9.00E-01	5.00E-05	5.00E-10	9.00E-01
	corx	corv	H-vol	corx	corv	H-vol	corx	corv	H-vol
Mean	8.62E-01	7.44E-02	1.28E-01	8.87E-01	4.29E-01	5.62E-01	8.88E-01	8.89E-01	8.94E-01
SD	1.04E-01	1.02E-01	6.68E-02	8.40E-02	8.42E-02	4.38E-02	8.13E-02	2.21E-02	1.16E-02
MSE	1.23E-02			7.24E-03			6.75E-03		
AVAR	1.94E-02			1.94E-02			1.94E-02		
n=5000	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx
True	5.00E-05	5.00E-07	9.00E-01	5.00E-05	5.00E-08	9.00E-01	5.00E-05	5.00E-10	9.00E-01
	corx	corv	H-vol	corx	corv	H-vol	corx	corv	H-vol
Mean	8.73E-01	4.80E-03	9.18E-03	8.95E-01	4.42E-02	8.20E-02	8.97E-01	7.53E-01	8.18E-01
SD	5.21E-02	3.30E-02	1.73E-02	3.80E-02	3.30E-02	1.67E-02	3.79E-02	1.44E-02	4.87E-03
MSE	3.43E-03			1.47E-03			1.45E-03		
AVAR	6.30E-03			6.30E-03			6.30E-03		
n=300	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx
True	5.00E-05	5.00E-07	-5.00E-01	5.00E-05	5.00E-08	-5.00E-01	5.00E-05	5.00E-10	-5.00E-01
	corx	corv	H-vol	corx	corv	H-vol	corx	corv	H-vol
Mean	-4.66E-01	-4.11E-02	-7.10E-02	-4.77E-01	-2.40E-01	-3.13E-01	-4.83E-01	-4.92E-01	-4.96E-01
SD	2.72E-01	1.01E-01	6.70E-02	2.68E-01	9.74E-02	5.48E-02	2.69E-01	7.94E-02	4.37E-02
MSE	7.49E-02			7.22E-02			7.24E-02		
AVAR	7.66E-02			7.66E-02			7.66E-02		
n=5000	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx	Σ_x	Σ_v	corx
True	5.00E-05	5.00E-07	-5.00E-01	5.00E-05	5.00E-08	-5.00E-01	5.00E-05	5.00E-10	-5.00E-01
	corx	corv	H-vol	corx	corv	H-vol	corx	corv	H-vol
Mean	-4.80E-01	-1.93E-03	-4.75E-03	-4.89E-01	-2.49E-02	-4.54E-02	-4.93E-01	-4.19E-01	-4.55E-01
SD	1.45E-01	3.31E-02	1.72E-02	1.41E-01	3.22E-02	1.66E-02	1.40E-01	2.79E-02	1.14E-02
MSE	2.13E-02			2.00E-02			1.96E-02		
AVAR	2.49E-02			2.49E-02			2.49E-02		

Data generating process:

$$y_{i,t} = x_{i,t} + v_{i,t}, i = 1,2$$

$$x_{i,t} = x_{i,t-1} + u_{i,t}$$

$$u_{i,t} \sim i.i.d.N(0, \sigma_x^2/n), v_{i,t} \sim i.i.d.N(0, \sigma_v^2)$$

$$corr(u_{1,t}, u_{2,t}) = corx$$

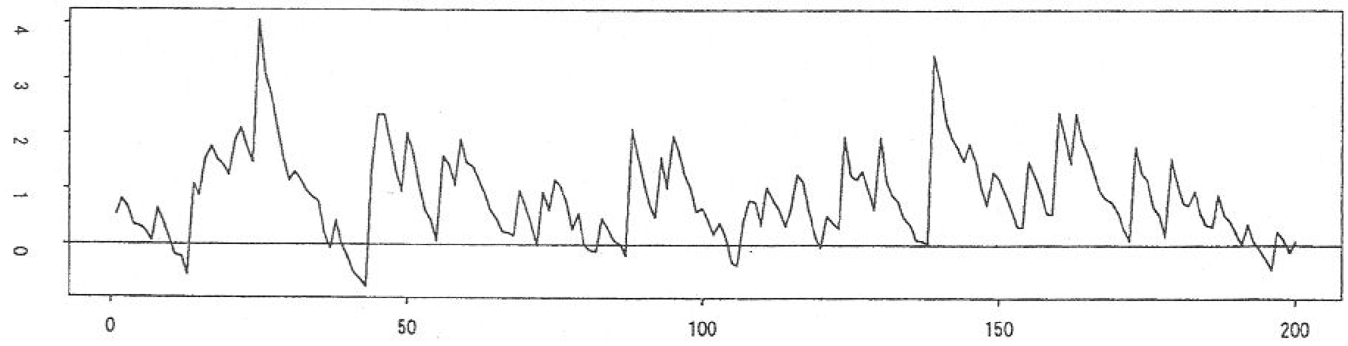
$$corr(v_{1,t}, v_{2,t}) = corv$$

SSAR Model

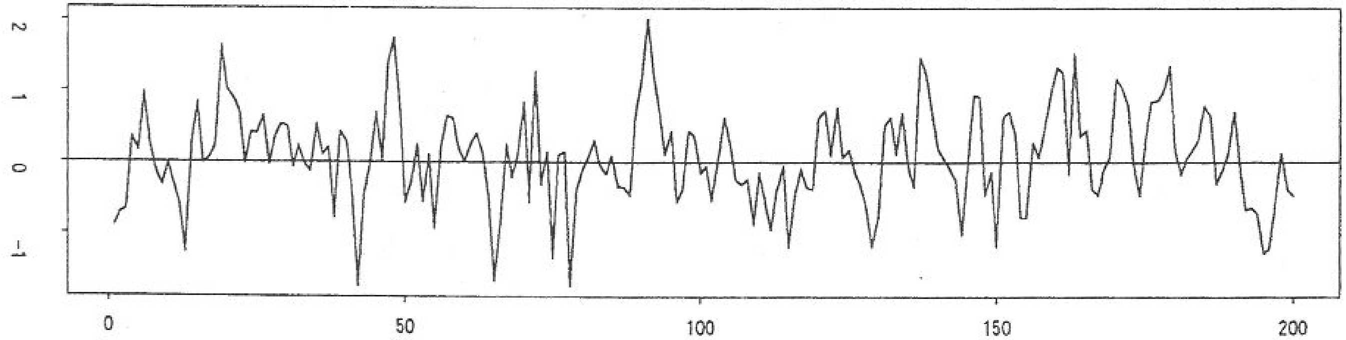
(Simultaneous Switching Autoregressive Model)

$$y_t = \begin{cases} a y_{t-1} + \sigma_1 \varepsilon_t & (y_t \geq y_{t-1}) \\ b y_{t-1} + \sigma_2 \varepsilon_t & (y_t < y_{t-1}) \end{cases}$$

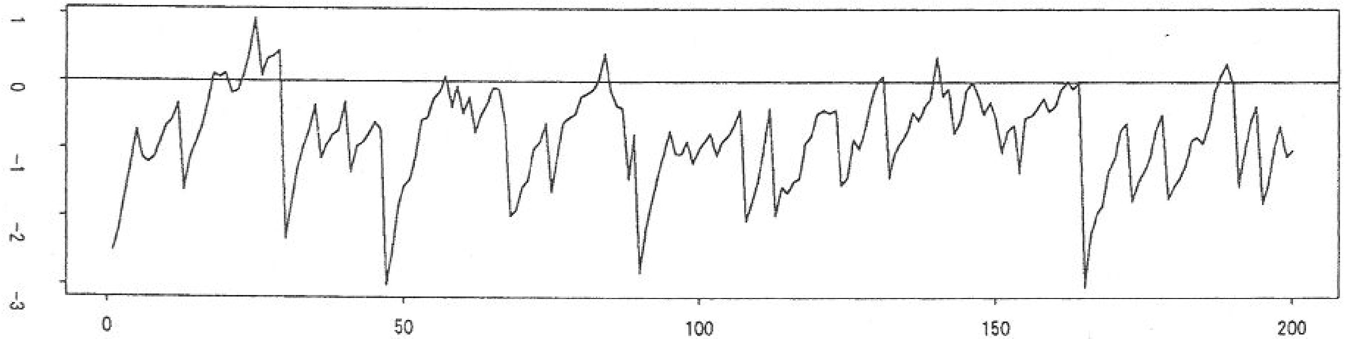
- Univariate case
- Switching Model
- Simultaneity (i.e. endogeneity)



A = -0.2, B = 0.8



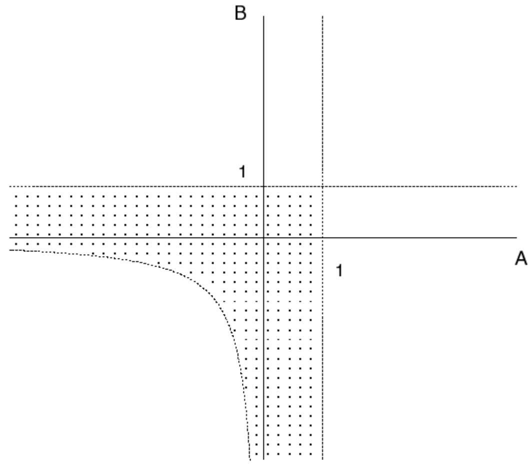
A = 0.4, B = 0.4



A = 0.8, B = -0.2

Stationarity condition for SSAR(1)(geometric ergodicity)

$$a < 1 \,,\ b < 1 \,,\ ab < 1$$



Remark: stationarity condition for AR(1)

$|a| < 1$

A derivation of non-stationary SSAR Model

$$\Delta P_t = g[V_t - P_{t-1}] + u_t$$

V : Intrinsic Value

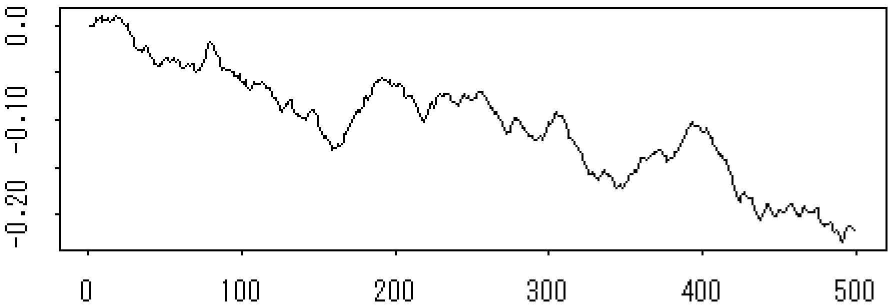
Then

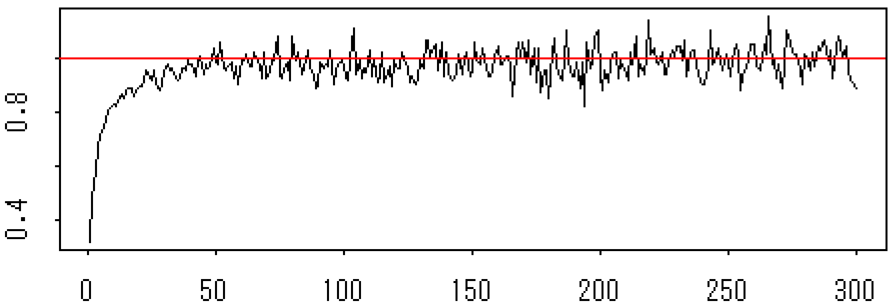
$$\Delta P_t = \begin{cases} g_1[V_t - P_{t-1}] & (V_t - P_{t-1} \geq 0) \\ g_2[V_t - P_{t-1}] & (V_t - P_{t-1} < 0) \end{cases}$$

$$\longrightarrow \quad P_t = \begin{cases} aP_{t-1} + \sigma_1 V_t & (P_t - P_{t-1} \geq 0) \\ bP_{t-1} + \sigma_2 V_t & (P_t - P_{t-1} < 0) \end{cases}$$

simulation

Model 1（a=0.5)





$$y_i = ay_{i-1} + (1-a)x_i$$

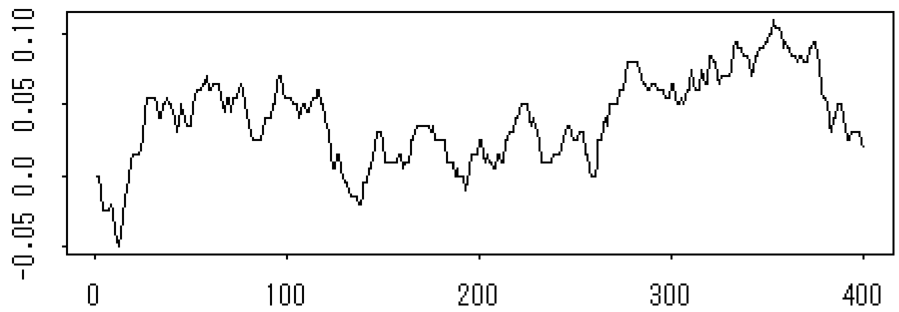
$$x_i = x_{i-1} + \varepsilon_i$$

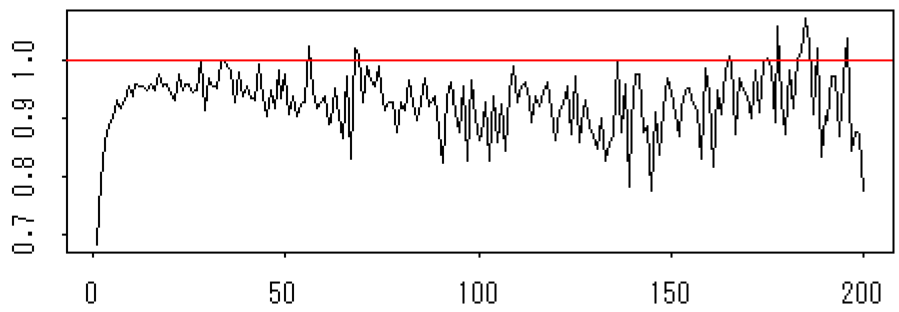
$$\varepsilon_i \sim i.i.d.N(0, \sigma_x^2 / N)$$

N=30000

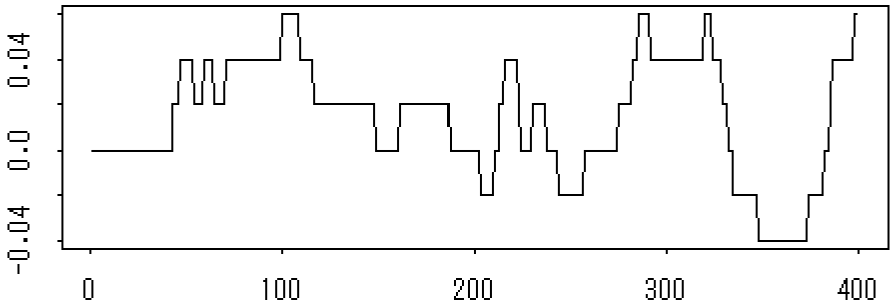
Model 2（Tick=0.005)

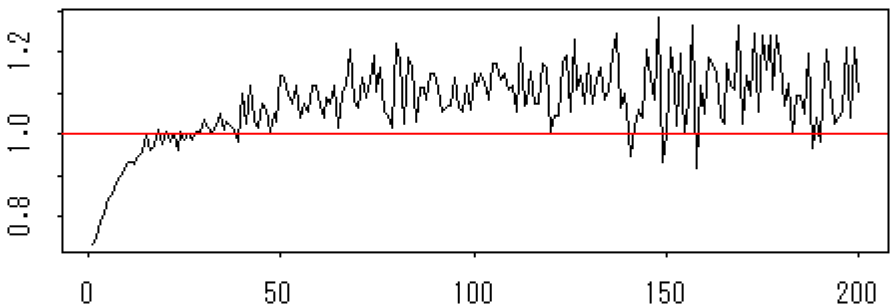
N=20000





Model 2（Tick=0.02)





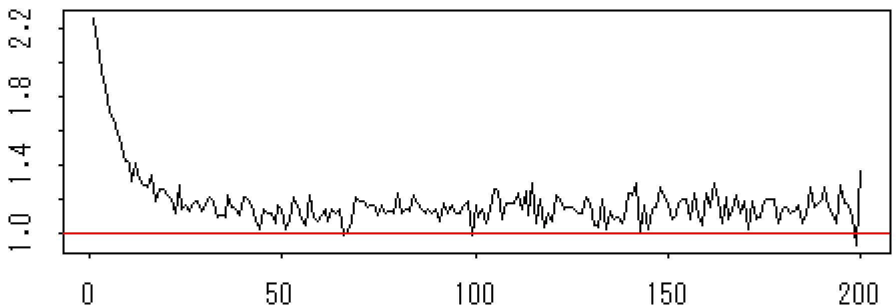
Integrated volatility（model 2）

work.sim1(1,0.0000,n=20000,nsim=1000)				a0=1,a1=0,a2=0	
0.038468				model2	thr=0.005
1.961663					
	sgx	sgv	H-vol	RK	
true-val	1	0.00E+00	1	1	
mean	1.002175	6.45E-06	0.685095	0.997254	
SD	0.194227	1.77E-07	0.008658	0.062124	
MSE	0.037729	4.16E-11	0.09924	0.003867	

work.sim1(15,0.01,n=20000,nsim=1000)				a0=7,a1=-12,a2=6	
107.3262				model2	thr=0.5
5710.043					
	sgx	sgv	H-vol	RK	
true-val	45	0.01	45	45	
mean	45.42377	0.005895	394.6072	61.89485	
SD	10.47896	0.000168	6.685363	4.065517	
MSE	109.9881	1.69E-05	122269.9	301.9644	

Model 3（g1=0.2,g2=5）

(SSAR:a=0.8,b=-4)



5-2 Simulations in Kunitomo-Misaki (2013)

(i) Basic Simulations

(ii) Extended Simulations

3-1 : Estimation of integrated volatility : Case 1 ($a_0 = 1, a_1 = a_2 = 0$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.06E-04	2.22E-04	2.07E-04	2.05E-04	2.04E-04	2.05E-04
		6.54E-05	4.52E-05	6.44E-05	8.34E-05	9.48E-05	1.31E-04
$\hat{\sigma}_v^2$	2.00E-06	2.03E-06	1.04E-07	9.82E-07	1.92E-06	2.18E-06	3.00E-06
		1.43E-07	6.00E-09	8.43E-08	2.19E-07	3.19E-07	8.30E-07
HI		7.39E-03	7.04E-03	4.74E-03	2.48E-03	1.40E-03	4.40E-04
		3.45E-04	3.35E-04	2.61E-04	1.79E-04	1.34E-04	8.46E-05
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.06E-04	2.07E-04	2.07E-04	2.07E-04	2.07E-04	2.06E-04
		4.12E-05	4.14E-05	6.55E-05	8.45E-05	9.66E-05	1.31E-04
$\hat{\sigma}_v^2$	2.00E-06	2.00E-06	9.40E-07	2.03E-06	2.09E-06	2.18E-06	3.00E-06
		5.54E-08	3.08E-08	1.41E-07	2.33E-07	3.19E-07	8.38E-07
HI		7.22E-02	4.57E-02	7.41E-03	2.60E-03	1.40E-03	4.40E-04
		1.06E-03	7.80E-04	2.96E-04	1.83E-04	1.35E-04	8.63E-05

3-2 : Estimation of integrated volatility: Case 2 ($a_0 = 1, a_1 = -1, a_2 = 1$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	1.67E-04	1.72E-04	1.89E-04	1.74E-04	1.71E-04	1.71E-04	1.72E-04
		5.52E-05	3.88E-05	5.44E-05	7.04E-05	8.01E-05	1.10E-04
$\hat{\sigma}_v^2$	2.00E-06	2.02E-06	1.04E-07	9.78E-07	1.90E-06	2.14E-06	2.83E-06
		1.43E-07	5.99E-09	8.40E-08	2.18E-07	3.15E-07	7.85E-07
HI		7.36E-03	7.01E-03	4.71E-03	2.44E-03	1.36E-03	4.06E-04
		3.44E-04	3.34E-04	2.60E-04	1.77E-04	1.32E-04	7.92E-05
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	1.67E-04	1.72E-04	1.73E-04	1.74E-04	1.74E-04	1.73E-04	1.73E-04
		3.50E-05	3.52E-05	5.56E-05	7.15E-05	8.16E-05	1.11E-04
$\hat{\sigma}_v^2$	2.00E-06	2.00E-06	9.39E-07	2.03E-06	2.08E-06	2.15E-06	2.84E-06
		5.54E-08	3.08E-08	1.41E-07	2.32E-07	3.14E-07	7.92E-07
HI		7.22E-02	4.57E-02	7.37E-03	2.57E-03	1.36E-03	4.07E-04
		1.06E-03	7.80E-04	2.95E-04	1.81E-04	1.33E-04	8.06E-05

3-3 : Estimation of integrated volatility: Case 3 (Stochastic Volatility)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.08E-04	2.23E-04	2.09E-04	2.07E-04	2.08E-04	2.11E-04
		6.94E-05	4.65E-05	6.78E-05	8.91E-05	1.07E-04	1.43E-04
$\hat{\sigma}_v^2$	2.00E-06	2.02E-06	1.04E-07	9.81E-07	1.93E-06	2.16E-06	3.03E-06
		1.42E-07	5.91E-09	8.50E-08	2.19E-07	3.19E-07	8.44E-07
HI		7.39E-03	7.04E-03	4.74E-03	2.48E-03	1.39E-03	4.45E-04
		3.49E-04	3.37E-04	2.52E-04	1.77E-04	1.33E-04	8.72E-05
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.06E-04	2.07E-04	2.08E-04	2.07E-04	2.08E-04	2.05E-04
		4.09E-05	4.10E-05	6.36E-05	8.17E-05	9.63E-05	1.29E-04
$\hat{\sigma}_v^2$	2.00E-06	2.00E-06	9.40E-07	2.02E-06	2.10E-06	2.19E-06	2.95E-06
		5.53E-08	3.04E-08	1.41E-07	2.27E-07	3.31E-07	8.17E-07
HI		7.22E-02	4.57E-02	7.40E-03	2.60E-03	1.40E-03	4.36E-04
		1.08E-03	7.96E-04	2.93E-04	1.79E-04	1.35E-04	8.61E-05

5-1 : Estimation of integrated volatility: Case 4 (Autoregressive Conditional Duration)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.03E-04	2.23E-04	2.06E-04	2.04E-04	2.03E-04	2.05E-04
		6.75E-05	4.50E-05	6.62E-05	8.41E-05	9.70E-05	1.33E-04
$\hat{\sigma}_v^2$	2.00E-06	2.03E-06	1.04E-07	9.75E-07	1.89E-06	2.17E-06	3.01E-06
		1.44E-07	8.05E-09	9.60E-08	2.18E-07	3.12E-07	8.29E-07
HI		7.41E-03	7.05E-03	4.70E-03	2.46E-03	1.39E-03	4.41E-04
		5.22E-04	4.85E-04	3.01E-04	1.78E-04	1.32E-04	8.47E-05
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.05E-04	2.06E-04	2.05E-04	2.04E-04	2.05E-04	2.07E-04
		4.14E-05	4.12E-05	6.49E-05	8.31E-05	9.73E-05	1.33E-04
$\hat{\sigma}_v^2$	2.00E-06	2.00E-06	9.31E-07	2.03E-06	2.10E-06	2.19E-06	3.00E-06
		5.56E-08	3.35E-08	1.41E-07	2.28E-07	3.18E-07	8.39E-07
HI		7.22E-02	4.52E-02	7.40E-03	2.61E-03	1.40E-03	4.40E-04
		1.67E-03	9.32E-04	2.95E-04	1.78E-04	1.34E-04	8.67E-05

5-2 : Estimation of integrated volatility: Case 5 ($g = 0.2$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.35E-04	4.22E-04	2.40E-04	2.16E-04	2.10E-04	2.08E-04
		7.42E-05	9.28E-05	7.44E-05	8.85E-05	9.76E-05	1.35E-04
$\hat{\sigma}_v^2$	2.00E-06	6.42E-07	5.67E-08	5.78E-07	1.73E-06	3.21E-06	6.31E-06
		4.58E-08	2.79E-09	4.75E-08	2.03E-07	4.79E-07	1.75E-06
HI		4.01E-03	3.98E-03	3.66E-03	3.07E-03	2.42E-03	8.33E-04
		1.66E-04	1.69E-04	1.89E-04	2.13E-04	2.26E-04	1.71E-04
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.27E-04	2.30E-04	2.10E-04	2.11E-04	2.10E-04	2.13E-04
		4.57E-05	4.60E-05	6.63E-05	8.60E-05	9.84E-05	1.35E-04
$\hat{\sigma}_v^2$	2.00E-06	6.28E-07	5.63E-07	4.32E-06	5.58E-06	5.72E-06	6.56E-06
		1.73E-08	1.74E-08	3.08E-07	6.24E-07	8.39E-07	1.83E-06
HI		4.00E-02	3.63E-02	1.74E-02	6.82E-03	3.51E-03	8.59E-04
		5.21E-04	5.86E-04	7.00E-04	4.85E-04	3.44E-04	1.79E-04

5-3 : Estimation of integrated volatility: Case 6 ($\eta = 0.001$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.05E-04	2.22E-04	2.06E-04	2.05E-04	2.04E-04	2.05E-04
		6.46E-05	4.55E-05	6.36E-05	8.45E-05	9.54E-05	1.34E-04
$\hat{\sigma}_v^2$	2.00E-06	2.11E-06	1.08E-07	1.02E-06	1.99E-06	2.25E-06	3.09E-06
		1.51E-07	6.26E-09	8.79E-08	2.29E-07	3.27E-07	8.52E-07
HI		7.68E-03	7.32E-03	4.93E-03	2.57E-03	1.44E-03	4.50E-04
		3.58E-04	3.47E-04	2.70E-04	1.86E-04	1.37E-04	8.62E-05
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.06E-04	2.07E-04	2.07E-04	2.08E-04	2.07E-04	2.08E-04
		4.08E-05	4.10E-05	6.53E-05	8.48E-05	9.73E-05	1.33E-04
$\hat{\sigma}_v^2$	2.00E-06	2.09E-06	9.78E-07	2.11E-06	2.17E-06	2.26E-06	3.10E-06
		5.76E-08	3.20E-08	1.47E-07	2.41E-07	3.29E-07	8.58E-07
HI		7.52E-02	4.76E-02	7.71E-03	2.70E-03	1.45E-03	4.51E-04
		1.10E-03	8.10E-04	3.06E-04	1.90E-04	1.41E-04	8.84E-05

5-4 : Estimation of integrated volatility: Case 7 ($\eta = 0.001$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.05E-04	2.23E-04	2.07E-04	2.05E-04	2.04E-04	2.05E-04
		6.46E-05	4.58E-05	6.36E-05	8.47E-05	9.57E-05	1.34E-04
$\hat{\sigma}_v^2$	2.00E-06	2.11E-06	1.09E-07	1.02E-06	1.99E-06	2.26E-06	3.09E-06
		1.48E-07	6.22E-09	8.75E-08	2.30E-07	3.30E-07	8.48E-07
HI		7.69E-03	7.33E-03	4.93E-03	2.57E-03	1.44E-03	4.50E-04
		3.53E-04	3.43E-04	2.69E-04	1.86E-04	1.38E-04	8.60E-05
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.06E-04	2.07E-04	2.07E-04	2.08E-04	2.07E-04	2.08E-04
		4.08E-05	4.10E-05	6.55E-05	8.51E-05	9.75E-05	1.33E-04
$\hat{\sigma}_v^2$	2.00E-06	2.09E-06	9.78E-07	2.11E-06	2.17E-06	2.26E-06	3.09E-06
		5.73E-08	3.20E-08	1.47E-07	2.38E-07	3.30E-07	8.53E-07
HI		7.52E-02	4.76E-02	7.71E-03	2.70E-03	1.45E-03	4.51E-04
		1.10E-03	8.09E-04	3.07E-04	1.89E-04	1.41E-04	8.79E-05

5-5 : Estimation of integrated volatility: Case 8 ($g_1 = 0.2, g_2 = 5$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.28E-04	3.12E-04	2.32E-04	2.23E-04	2.22E-04	2.25E-04
		7.27E-05	6.87E-05	7.29E-05	9.22E-05	1.05E-04	1.46E-04
$\hat{\sigma}_v^2$	2.00E-06	2.46E-06	1.70E-07	1.69E-06	4.29E-06	5.93E-06	7.08E-06
		2.11E-07	1.46E-08	1.79E-07	5.65E-07	9.76E-07	2.26E-06
HI		1.21E-02	1.17E-02	9.32E-03	6.06E-03	3.72E-03	9.28E-04
		9.54E-04	9.43E-04	8.27E-04	6.15E-04	4.61E-04	2.36E-04
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
$\hat{\sigma}_x^2$	2.00E-04	2.24E-04	2.27E-04	2.23E-04	2.25E-04	2.23E-04	2.27E-04
		4.62E-05	4.69E-05	7.21E-05	9.28E-05	1.05E-04	1.45E-04
$\hat{\sigma}_v^2$	2.00E-06	2.35E-06	1.60E-06	5.85E-06	5.88E-06	5.97E-06	6.82E-06
		7.32E-08	6.13E-08	4.46E-07	7.24E-07	9.81E-07	2.14E-06
HI		1.16E-01	8.90E-02	2.11E-02	7.15E-03	3.67E-03	8.98E-04
		3.01E-03	2.55E-03	1.09E-03	6.43E-04	4.56E-04	2.21E-04

3-1 : Estimation of hedging coefficent: Case 1 ($a_0 = 1, a_1 = a_2 = 0; \lambda = 1800$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	1.00E-04	6.75E-05	7.33E-06	4.30E-05	6.34E-05	8.03E-05	9.53E-05
		1.78E-04	5.34E-05	1.24E-04	1.19E-04	9.33E-05	6.15E-05
HY	1.00E-04	1.05E-04	(1.25E-04)				
RCV-RV	5.00E-01	9.08E-03	1.04E-03	8.98E-03	2.55E-02	5.77E-02	2.20E-01
		2.41E-02	7.57E-03	2.60E-02	4.83E-02	6.68E-02	1.39E-01
HY-RV	5.00E-01	1.42E-02	1.49E-02	2.21E-02	4.24E-02	7.59E-02	2.49E-01
		1.70E-02	1.78E-02	2.64E-02	5.06E-02	9.11E-02	3.04E-01
HY-SIML	5.00E-01	5.81E-01	4.87E-01	5.71E-01	6.31E-01	6.86E-01	8.72E-01
		7.54E-01	5.98E-01	7.42E-01	8.74E-01	1.06E+00	1.75E+00
SIML-SIML	5.00E-01	4.97E-01	4.51E-01	4.91E-01	5.01E-01	5.17E-01	5.05E-01
		2.42E-01	1.29E-01	2.05E-01	2.72E-01	3.30E-01	5.11E-01
RCV	1.00E-04	6.67E-05	4.92E-06	3.73E-05	6.84E-05	8.35E-05	9.76E-05
		8.61E-06	2.17E-06	6.10E-06	9.70E-06	1.37E-05	3.01E-05
HY	1.00E-04	1.00E-04	(1.11E-05)				
1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV-RV	5.00E-01	2.45E-01	1.83E-02	1.52E-01	3.08E-01	3.96E-01	4.82E-01
		2.96E-02	8.08E-03	2.40E-02	3.97E-02	5.52E-02	1.16E-01
HY-RV	5.00E-01	3.68E-01	3.73E-01	4.09E-01	4.51E-01	4.77E-01	5.12E-01
		3.76E-02	3.81E-02	4.17E-02	4.62E-02	5.22E-02	1.06E-01
HY-SIML	5.00E-01	5.69E-01	5.22E-01	5.65E-01	6.10E-01	6.65E-01	9.02E-01
		2.10E-01	1.18E-01	2.01E-01	2.98E-01	4.31E-01	1.21E+00
SIML-SIML	5.00E-01	5.10E-01	5.01E-01	5.11E-01	5.13E-01	5.29E-01	5.24E-01
		2.39E-01	1.23E-01	2.03E-01	2.71E-01	3.36E-01	5.02E-01

3-2 : Estimation of hedging coeffcient: Case 1 ($a_0 = 1, a_1 = a_2 = 0; \lambda = 18000$)

18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	1.00E-04	8.27E-05	4.15E-05	1.02E-04	1.02E-04	1.03E-04	1.02E-04
		5.44E-04	3.83E-04	2.10E-04	1.27E-04	9.32E-05	6.32E-05
HY	1.00E-04	9.94E-05	(3.86E-04)				
RCV-RV	5.00E-01	1.14E-03	9.13E-04	1.38E-02	3.91E-02	7.36E-02	2.31E-01
		7.53E-03	8.39E-03	2.83E-02	4.87E-02	6.60E-02	1.37E-01
HY-RV	5.00E-01	1.38E-03	2.18E-03	1.35E-02	3.78E-02	7.09E-02	2.32E-01
		5.35E-03	8.45E-03	5.23E-02	1.49E-01	2.78E-01	9.25E-01
HY-SIML	5.00E-01	5.11E-01	5.07E-01	5.20E-01	5.31E-01	5.70E-01	9.00E-01
		2.00E+00	1.99E+00	2.24E+00	2.52E+00	2.89E+00	5.28E+00
SIML-SIML	5.00E-01	4.91E-01	4.86E-01	4.99E-01	5.12E-01	5.18E-01	5.19E-01
		1.44E-01	1.30E-01	2.02E-01	2.68E-01	3.17E-01	5.16E-01
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	1.00E-04	6.67E-05	3.68E-05	9.00E-05	9.67E-05	9.82E-05	1.00E-04
		6.77E-06	5.28E-06	6.86E-06	9.94E-06	1.37E-05	2.98E-05
HY	1.00E-04	9.99E-05	(6.36E-06)				
RCV-RV	5.00E-01	7.25E-02	5.62E-02	3.31E-01	4.32E-01	4.63E-01	4.95E-01
		7.34E-03	8.04E-03	2.27E-02	3.70E-02	5.18E-02	1.18E-01
HY-RV	5.00E-01	1.09E-01	1.53E-01	3.68E-01	4.47E-01	4.74E-01	5.11E-01
		6.90E-03	9.59E-03	2.45E-02	3.62E-02	4.67E-02	1.00E-01
HY-SIML	5.00E-01	5.22E-01	5.23E-01	5.57E-01	5.93E-01	6.39E-01	8.14E-01
		1.13E-01	1.13E-01	1.98E-01	2.98E-01	4.04E-01	9.23E-01
SIML-SIML	5.00E-01	5.05E-01	5.02E-01	5.15E-01	5.29E-01	5.33E-01	5.40E-01
		1.40E-01	1.25E-01	1.99E-01	2.63E-01	3.12E-01	4.85E-01

3-3 : Estimation of hedging coefficient: Case 2 ($a_0 = 1, a_1 = -1, a_2 = 1; \lambda = 1800$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV-RV	5.00E-01	7.60E-03	9.30E-04	7.71E-03	2.12E-02	4.90E-02	1.98E-01
		2.40E-02	7.57E-03	2.61E-02	4.85E-02	6.73E-02	1.40E-01
HY-RV	5.00E-01	1.20E-02	1.26E-02	1.87E-02	3.61E-02	6.53E-02	2.26E-01
		1.69E-02	1.77E-02	2.63E-02	5.08E-02	9.24E-02	3.24E-01
HY-SIML	5.00E-01	5.83E-01	4.79E-01	5.72E-01	6.37E-01	6.96E-01	8.68E-01
		8.82E-01	6.95E-01	8.66E-01	1.02E+00	1.23E+00	1.98E+00
SIML-SIML	5.00E-01	4.94E-01	4.43E-01	4.88E-01	4.98E-01	5.14E-01	5.02E-01
		2.44E-01	1.31E-01	2.08E-01	2.74E-01	3.31E-01	5.13E-01
1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	8.33E-04	5.56E-05	4.10E-06	3.11E-05	5.70E-05	6.96E-05	8.14E-05
		7.39E-06	1.88E-06	5.26E-06	8.24E-06	1.15E-05	2.51E-05
HY	8.33E-04	8.35E-05	(9.40E-06)				
RCV-RV	5.00E-01	2.33E-01	1.74E-02	1.47E-01	3.01E-01	3.92E-01	4.81E-01
		2.91E-02	8.02E-03	2.40E-02	3.98E-02	5.54E-02	1.16E-01
HY-RV	5.00E-01	3.50E-01	3.55E-01	3.94E-01	4.42E-01	4.71E-01	5.10E-01
		3.66E-02	3.71E-02	4.11E-02	4.62E-02	5.23E-02	1.07E-01
HY-SIML	5.00E-01	5.69E-01	5.22E-01	5.65E-01	6.11E-01	6.67E-01	9.04E-01
		2.11E-01	1.18E-01	2.03E-01	3.02E-01	4.34E-01	1.21E+00
SIML-SIML	5.00E-01	5.09E-01	5.01E-01	5.10E-01	5.12E-01	5.28E-01	5.23E-01
		2.40E-01	1.24E-01	2.04E-01	2.72E-01	3.35E-01	5.04E-01

3-4 : Estimation of hedging coefficient: Case 2 $(a_0 = 1, a_1 = -1, a_2 = 1; \lambda = 18000)$

18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	8.33E-04	7.16E-05	3.55E-05	8.70E-05	8.57E-05	8.71E-05	8.51E-05
		5.44E-04	3.83E-04	2.09E-04	1.25E-04	9.14E-05	5.88E-05
HY	8.33E-04	8.28E-05	(3.86E-04)				
RCV-RV	5.00E-01	9.90E-04	7.81E-04	1.18E-02	3.33E-02	6.35E-02	2.09E-01
		7.53E-03	8.39E-03	2.84E-02	4.89E-02	6.64E-02	1.39E-01
HY-RV	5.00E-01	1.15E-03	1.81E-03	1.13E-02	3.18E-02	6.04E-02	2.09E-01
		5.35E-03	8.45E-03	5.26E-02	1.51E-01	2.85E-01	1.00E+00
HY-SIML	5.00E-01	5.09E-01	5.04E-01	5.13E-01	5.20E-01	5.60E-01	9.17E-01
		2.39E+00	2.37E+00	2.69E+00	3.02E+00	3.47E+00	6.06E+00
SIML-SIML	5.00E-01	4.89E-01	4.83E-01	4.96E-01	5.08E-01	5.15E-01	5.18E-01
		1.47E-01	1.32E-01	2.05E-01	2.72E-01	3.21E-01	5.20E-01
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV-RV	5.00E-01	6.27E-02	4.93E-02	3.15E-01	4.23E-01	4.58E-01	4.94E-01
		7.32E-03	8.08E-03	2.29E-02	3.72E-02	5.20E-02	1.19E-01
HY-RV	5.00E-01	9.39E-02	1.34E-01	3.50E-01	4.38E-01	4.68E-01	5.10E-01
		6.65E-03	9.40E-03	2.56E-02	3.80E-02	4.84E-02	1.02E-01
HY-SIML	5.00E-01	5.23E-01	5.23E-01	5.58E-01	5.94E-01	6.40E-01	8.17E-01
		1.14E-01	1.14E-01	2.00E-01	3.00E-01	4.04E-01	9.37E-01
SIML-SIML	5.00E-01	5.05E-01	5.02E-01	5.15E-01	5.28E-01	5.32E-01	5.40E-01
		1.42E-01	1.26E-01	2.01E-01	2.65E-01	3.15E-01	4.87E-01

3-5 : Estimation of hedging coefficient: Case 3 (Stochastic volatility; $\lambda = 1800$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV		7.23E-05	4.27E-06	3.79E-05	6.44E-05	8.49E-05	9.93E-05
		1.74E-04	5.33E-05	1.24E-04	1.20E-04	9.15E-05	6.27E-05
HY		9.83E-05	(1.22E-04)				
RCV-RV	5.00E-01	9.78E-03	6.24E-04	7.99E-03	2.61E-02	6.18E-02	2.28E-01
		2.35E-02	7.57E-03	2.61E-02	4.85E-02	6.64E-02	1.40E-01
HY-RV	5.00E-01	1.33E-02	1.40E-02	2.07E-02	3.99E-02	7.16E-02	2.31E-01
		1.65E-02	1.73E-02	2.58E-02	4.97E-02	8.97E-02	3.01E-01
HY-SIML	5.00E-01	5.34E-01	4.58E-01	5.23E-01	5.86E-01	6.25E-01	8.47E-01
		7.17E-01	5.96E-01	6.97E-01	8.44E-01	9.46E-01	1.76E+00
SIML-SIML	5.00E-01	5.03E-01	4.53E-01	5.01E-01	5.11E-01	5.08E-01	5.02E-01
		2.41E-01	1.30E-01	2.06E-01	2.85E-01	3.46E-01	5.11E-01

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV		6.69E-05	4.79E-06	3.67E-05	6.83E-05	8.35E-05	9.73E-05
		8.82E-06	2.22E-06	6.09E-06	9.65E-06	1.32E-05	2.83E-05
HY		1.00E-04	(1.13E-05)				
RCV-RV	5.00E-01	2.46E-01	1.78E-02	1.50E-01	3.07E-01	3.96E-01	4.81E-01
		3.06E-02	8.24E-03	2.42E-02	3.99E-02	5.39E-02	1.13E-01
HY-RV	5.00E-01	3.68E-01	3.73E-01	4.08E-01	4.51E-01	4.76E-01	5.09E-01
		3.76E-02	3.81E-02	4.15E-02	4.58E-02	5.26E-02	1.02E-01
HY-SIML	5.00E-01	5.68E-01	5.20E-01	5.64E-01	6.18E-01	6.50E-01	8.19E-01
		2.17E-01	1.15E-01	2.09E-01	3.25E-01	3.83E-01	8.85E-01
SIML-SIML	5.00E-01	5.15E-01	4.99E-01	5.17E-01	5.21E-01	5.17E-01	5.16E-01
		2.39E-01	1.23E-01	2.01E-01	2.74E-01	3.34E-01	5.08E-01

3-6 : Estimation of coefficient coefficient: Case 3 (Stochastic volatility; $\lambda = 18000$)

18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV		8.23E-05	4.50E-05	7.87E-05	8.71E-05	9.35E-05	9.86E-05
		5.36E-04	3.80E-04	2.10E-04	1.25E-04	9.46E-05	6.15E-05
HY		1.31E-04	(3.86E-04)				
RCV-RV	5.00E-01	1.14E-03	9.88E-04	1.07E-02	3.37E-02	6.71E-02	2.25E-01
		7.42E-03	8.31E-03	2.84E-02	4.86E-02	6.79E-02	1.37E-01
HY-RV	5.00E-01	1.81E-03	2.86E-03	1.76E-02	5.05E-02	9.24E-02	3.11E-01
		5.34E-03	8.43E-03	5.20E-02	1.49E-01	2.78E-01	9.22E-01
HY-SIML	5.00E-01	6.79E-01	6.76E-01	7.17E-01	7.83E-01	8.27E-01	1.04E+00
		2.01E+00	2.01E+00	2.18E+00	2.51E+00	2.83E+00	4.02E+00
SIML-SIML	5.00E-01	4.93E-01	4.89E-01	4.98E-01	4.98E-01	4.95E-01	4.78E-01
		1.40E-01	1.26E-01	1.97E-01	2.68E-01	3.21E-01	4.85E-01
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV		6.68E-05	3.70E-05	8.99E-05	9.64E-05	9.84E-05	1.01E-04
		7.08E-06	5.24E-06	6.89E-06	9.77E-06	1.34E-05	2.86E-05
HY		1.00E-04	(6.32E-06)				
RCV-RV	5.00E-01	7.26E-02	5.65E-02	3.31E-01	4.31E-01	4.65E-01	4.94E-01
		7.74E-03	8.00E-03	2.29E-02	3.65E-02	5.14E-02	1.11E-01
HY-RV	5.00E-01	1.09E-01	1.53E-01	3.70E-01	4.50E-01	4.77E-01	5.09E-01
		6.89E-03	9.63E-03	2.45E-02	3.55E-02	4.75E-02	9.81E-02
HY-SIML	5.00E-01	5.21E-01	5.21E-01	5.51E-01	5.92E-01	6.28E-01	8.22E-01
		1.11E-01	1.10E-01	1.90E-01	2.75E-01	3.66E-01	7.92E-01
SIML-SIML	5.00E-01	5.05E-01	5.03E-01	5.12E-01	5.12E-01	5.11E-01	5.04E-01
		1.38E-01	1.22E-01	1.92E-01	2.64E-01	3.19E-01	4.99E-01

Table 5.1 : Estimation of hedging coefficient: Case 4 (ACD; $\lambda = 1800$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	1.00E-04	6.82E-05	3.41E-06	3.80E-05	7.20E-05	8.57E-05	9.75E-05
		1.72E-04	5.30E-05	1.25E-04	1.24E-04	9.39E-05	6.17E-05
HY	1.00E-04	1.05E-04					
		1.23E-04					
RCV-RV	5.00E-01	9.13E-03	4.78E-04	8.08E-03	2.94E-02	6.17E-02	2.24E-01
		2.32E-02	7.47E-03	2.64E-02	5.03E-02	6.74E-02	1.38E-01
HY-RV	5.00E-01	1.42E-02	1.49E-02	2.24E-02	4.32E-02	7.68E-02	2.51E-01
		1.66E-02	1.75E-02	2.62E-02	5.06E-02	8.99E-02	3.00E-01
HY-SIML	5.00E-01	5.90E-01	4.92E-01	5.78E-01	6.48E-01	6.87E-01	8.83E-01
		7.53E-01	5.98E-01	7.33E-01	9.13E-01	1.05E+00	1.74E+00
SIML-SIML	5.00E-01	5.07E-01	4.50E-01	4.96E-01	5.06E-01	5.16E-01	4.94E-01
		2.46E-01	1.28E-01	2.08E-01	2.73E-01	3.23E-01	4.92E-01
1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	1.00E-04	6.86E-05	4.69E-06	3.60E-05	6.72E-05	8.27E-05	9.80E-05
		9.08E-06	2.15E-06	6.40E-06	9.82E-06	1.30E-05	2.82E-05
HY	1.00E-04	1.01E-04					
		1.12E-05					
RCV-RV	5.00E-01	2.51E-01	1.74E-02	1.47E-01	3.02E-01	3.91E-01	4.83E-01
		3.18E-02	7.98E-03	2.51E-02	4.04E-02	5.32E-02	1.11E-01
HY-RV	5.00E-01	3.68E-01	3.74E-01	4.10E-01	4.52E-01	4.77E-01	5.10E-01
		3.82E-02	3.86E-02	4.14E-02	4.69E-02	5.47E-02	1.04E-01
HY-SIML	5.00E-01	5.69E-01	5.22E-01	5.65E-01	6.14E-01	6.59E-01	8.82E-01
		2.09E-01	1.17E-01	2.04E-01	3.10E-01	4.18E-01	1.07E+00
SIML-SIML	5.00E-01	5.19E-01	5.02E-01	5.16E-01	5.24E-01	5.34E-01	5.19E-01
		2.45E-01	1.21E-01	2.05E-01	2.77E-01	3.25E-01	5.13E-01

Table 5.2 : Estimation of hedging coefficient: Case 4 (ACD ; $\lambda = 18000$)

18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	1.00E-04	7.51E-05	3.09E-05	8.39E-05	9.73E-05	1.04E-04	1.00E-04
		5.01E-04	3.78E-04	2.16E-04	1.26E-04	9.30E-05	6.06E-05
HY	1.00E-04	1.31E-04					
		3.80E-04					
RCV-RV	5.00E-01	1.04E-03	6.88E-04	1.13E-02	3.75E-02	7.38E-02	2.32E-01
		6.94E-03	8.36E-03	2.91E-02	4.86E-02	6.60E-02	1.39E-01
HY-RV	5.00E-01	1.81E-03	2.90E-03	1.78E-02	5.08E-02	9.60E-02	3.10E-01
		5.26E-03	8.41E-03	5.14E-02	1.48E-01	2.72E-01	9.22E-01
HY-SIML	5.00E-01	6.77E-01	6.76E-01	7.16E-01	8.01E-01	8.07E-01	1.03E+00
		2.00E+00	1.98E+00	2.14E+00	2.47E+00	2.68E+00	4.58E+00
SIML-SIML	5.00E-01	4.90E-01	4.85E-01	4.98E-01	5.09E-01	5.21E-01	5.24E-01
		1.46E-01	1.28E-01	2.06E-01	2.75E-01	3.46E-01	5.14E-01
18000	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	1.00E-04	6.84E-05	3.59E-05	8.92E-05	9.69E-05	9.93E-05	1.01E-04
		6.91E-06	5.18E-06	6.92E-06	1.02E-05	1.34E-05	2.88E-05
HY	1.00E-04	1.00E-04					
		6.34E-06					
RCV-RV	5.00E-01	7.44E-02	5.53E-02	3.28E-01	4.31E-01	4.67E-01	5.01E-01
		7.53E-03	7.97E-03	2.32E-02	3.73E-02	5.14E-02	1.14E-01
HY-RV	5.00E-01	1.09E-01	1.55E-01	3.70E-01	4.49E-01	4.76E-01	5.16E-01
		7.11E-03	9.88E-03	2.53E-02	3.80E-02	4.97E-02	1.02E-01
HY-SIML	5.00E-01	5.25E-01	5.26E-01	5.60E-01	6.15E-01	6.58E-01	8.42E-01
		1.16E-01	1.16E-01	1.93E-01	3.17E-01	4.24E-01	8.77E-01
SIML-SIML	5.00E-01	5.03E-01	5.02E-01	5.13E-01	5.25E-01	5.30E-01	5.35E-01
		1.42E-01	1.25E-01	1.99E-01	2.68E-01	3.29E-01	4.94E-01

Table 5.3 : Estimation of hedging coefficient: Case 5 ($g = 0.2; \lambda = 1800$)

1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	1.00E-04	9.97E-06	2.88E-07	7.30E-06	2.12E-05	4.27E-05	8.36E-05
		1.03E-04	2.96E-05	8.65E-05	1.34E-04	1.55E-04	1.21E-04
HY	1.00E-04	1.74E-05					
		1.16E-04					
RCV-RV	5.00E-01	2.46E-03	9.30E-05	1.98E-03	6.85E-03	1.76E-02	1.01E-01
		2.57E-02	7.48E-03	2.37E-02	4.37E-02	6.41E-02	1.48E-01
HY-RV	5.00E-01	4.32E-03	4.35E-03	4.74E-03	5.56E-03	6.97E-03	2.12E-02
		2.89E-02	2.91E-02	3.17E-02	3.78E-02	4.83E-02	1.47E-01
HY-SIML	5.00E-01	7.18E-02	4.00E-02	6.80E-02	7.64E-02	1.08E-01	1.45E-01
		6.02E-01	3.01E-01	5.87E-01	7.59E-01	9.28E-01	1.54E+00
SIML-SIML	5.00E-01	4.25E-01	2.26E-01	4.11E-01	4.72E-01	4.98E-01	4.92E-01
		2.45E-01	1.54E-01	2.23E-01	2.83E-01	3.34E-01	5.17E-01
1800	True	Raw	1 sec.	10 sec.	30 sec.	60 sec.	300 sec.
RCV	1.00E-04	1.34E-05	9.91E-07	9.43E-06	2.49E-05	4.19E-05	8.38E-05
		2.48E-06	4.92E-07	1.99E-06	4.52E-06	7.85E-06	2.59E-05
HY	1.00E-04	2.01E-05					
		3.54E-06					
RCV-RV	5.00E-01	2.15E-01	1.56E-02	1.26E-01	2.58E-01	3.51E-01	4.75E-01
		3.73E-02	7.73E-03	2.48E-02	4.16E-02	5.69E-02	1.16E-01
HY-RV	5.00E-01	3.22E-01	3.16E-01	2.67E-01	2.09E-01	1.68E-01	1.17E-01
		5.28E-02	5.14E-02	4.27E-02	3.28E-02	2.63E-02	2.44E-02
HY-SIML	5.00E-01	1.14E-01	1.09E-01	1.13E-01	1.22E-01	1.33E-01	1.80E-01
		4.21E-02	2.48E-02	4.01E-02	5.97E-02	8.67E-02	2.41E-01
SIML-SIML	5.00E-01	5.10E-01	4.93E-01	5.09E-01	5.13E-01	5.29E-01	5.23E-01
		2.39E-01	1.24E-01	2.04E-01	2.71E-01	3.34E-01	4.97E-01

6 Conclusion

1. The SIML estimator is simple and it has reasonable statistical properties.
2. We have the asymptotic robustness in the sense that it is consistent and it has the asymptotic normality (in a proper sense) under a fairly general conditions. They include not only the cases when the micro-market noises are possibly autocorrelated, we have non-linear price adjustments including the round-off errors and the high-frequency data are randomly sampled.
3. The SIML estimator is also simple and useful for multivariate high frequency series including the estimation of integrated covariances and the hedging coefficient.

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