Dynamic Hedging of Counterparty Exposure

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September 14, 2010

Key words Markov Copula, Counterparty Risk, Collateralization, Credit Risk, Credit Valuation Adjustment, Expected Positive Exposure, Markov Copula, Joint Defaults, Hedging.

We consider the issue of counterparty credit risk valuation and hedging. Counterparty risk in general is the risk that a party to an OTC derivative contract may fail to perform on its contractual obligations, causing losses to the other party. A major issue in this regard is the so-called *wrong way risk*, namely the risk that the value of the contract is particularly high from the perspective of the other party at the moment of default of the counterparty. Since the sub-prime crisis, counterparty risk and wrong way risk are a crucial issue in connection with valuation and risk management of credit derivatives. We first derive a general representation for the Credit Valuation Adjustment (CVA, namely the correction in value accounting for counterparty risk) of a portfolio of OTC derivatives, with netting and collateralization, and a general equation for the dynamics of the CVA.

The previous results are essentially model free. However they do not immediately lend themselves to any practical computations. We therefore subsequently them to the counterparty risk of a portfolio of credit derivatives including, for instance, CDSs and/or CDOs, considered in a suitable Markovian model. Wrong way risk is represented in the model by the possibility of simultaneous defaults. Moreover a common shocks interpretation of the model is possible so that efficient convolution recursion procedures are available for pricing portfolio loss derivatives, conditionally on any given state of the Markov model. These pricing procedures allow one to derive an exact Monte Carlo scheme for the CVA of the portfolio. By exact we mean that the Monte Carlo involves no time-discretisation error, nor any approximation error regarding the mark-to-market of the portfolio at the time of default of the counterparty. The only error left is thus the statistical Monte Carlo error.

We then deal with the issue of dynamic hedging of the CVA in the Markov model. We establish a rigorous connection between the CVA, which represents the price of the counterparty risk, and a suitable notion of Expected Positive Exposure (EPE). Specifically, the EPE emerges as the key

ingredient of the min-variance hedging ratio of the CVA by a CDS on the counterparty. Related notions of EPE have actually long been used in an ad-hoc way by practitioners for hedging their CVA. Our analysis thus justifies rigorously this market practice, making also precise the proper definition of the EPE which should be used in this regard, and the way in which the EPE should intervene in the hedging strategy.

References

[1] Assefa, S., Bielecki, T. R. and Crepey, S.: Dynamic Hedging of Counterparty Exposure. *In Preparation.*