From default to recovery and to (economic) default

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Dec 17, 2010

Outline

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Two key quantities in credit risk

- Default processes: the probability of default
- Recovery rate process: the salvage value in case of default

Modeling Default

- Structural approach: model default via the dynamics of a firm's asset value process $S = (S_t)_{t \geq 0}$ and debt induced level D,
$$\tau^* = \inf\{t > 0 : S_t \leq D\}.$$
Merton(1974), Black-Cox (1976), Leland (1994), Leland and Toft (1996)
- Reduced-form approach: model the intensity of the arrival of default by an exogenous random process λ_t.
Books: *Ammann (2001), Bielecki and Rutkowski (2002), Duffie and Singleton (2003), Lando (2004)*

Two approaches are one

- From a finance view point, the two approaches are related to the information set available to the management or to the market or by adding uncertainty;
- From a probabilistic viewpoint, reduced-form models can be obtained from structural models by conditioning on the smaller information set available to the market or by introducing additional random processes.

Outline

- Some Background
  - Default Risk and Existing Approaches
  - Recovery Rate
  - Arcsine Law
- Economic Default
  - Why Economic Default
  - Modeling Default τ_r and Economic Default τ_e
  - Analyzing τ_r and τ_e
  - Relation to Existing Approaches
- Numerical Example
- Concluding Remarks

Recovery rate estimate

- Recovery of Face Value (RFV):

$$B^d_\tau = \delta_\tau F$$

- Recovery of Treasury (RT)

$$B^d_\tau = \delta_\tau p(\tau, T)$$

- Recovery of Market Value (RMV)

$$B^d_\tau = \delta_\tau B_{\tau^-}$$

- Moody's cross-sectional approach for RFV ($= 0.422$).

Outline

Arcsine Law

- (P. Lévy) Arcsine Law for the occupation time of Brownian motion W on the positive half line: for $0 \leq \alpha \leq t$,

$$P_r\left(\int_0^t 1_{(0,\infty)}(W_s)ds \leq \alpha\right) = \int_0^{\frac{\alpha}{t}} \frac{ds}{\pi\sqrt{s(1-s)}} = \frac{2}{\pi}\arcsin\left(\frac{\alpha}{t}\right)^{\frac{1}{2}},$$

- Let $\theta_t = \sup\{0 \leq s \leq t : W_s = \max_{0\leq s\leq t} W_s\}$, then

$$P(\theta_t \in ds) = \frac{ds}{\pi\sqrt{s(1-s)}}.$$

- Let $\gamma_t = \sup\{0 \leq s \leq t : W_s = 0\}$, then

$$P(\gamma_t \in ds) = \frac{ds}{\pi\sqrt{s(1-s)}}.$$

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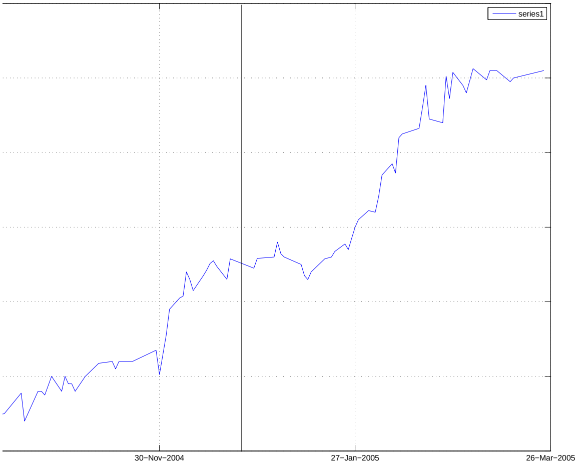
Reality check: “data mining” from the market

An empirical study by Guo, Jarrow and Lin (2008) on 2500 defaulted bonds from a datebase with 20 million price quotes over 31000 different issues showed that

- the market anticipates the default before it actually occurs;
- accurate default date is crucial to the recovery rate estimate.

Example: Trico Marine



Market anticipates default

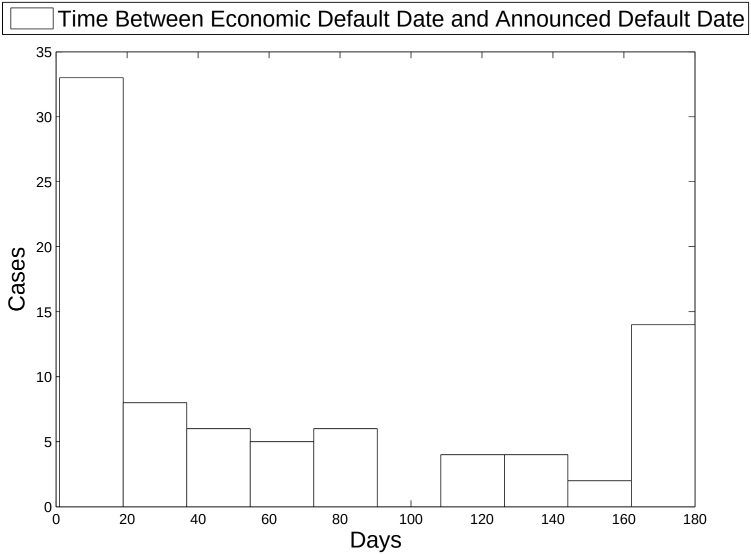
Difference	Count	Avg. Price	Std. Dev.	Avg. Ratio
-30	23	62.19	33.04	1.3199
-20	58	48.99	28.06	1.3510
-15	46	54.70	29.46	1.2299
-10	27	66.74	28.60	1.2382
-7	60	48.31	26.77	1.1574
-5	51	40.42	25.81	1.2114
-3	29	55.33	29.50	1.0639
-2	41	44.60	29.99	1.0541
-1	61	45.05	29.55	0.9796
0	70	48.17	29.39	1
1	71	45.48	28.67	1.0292
2	63	41.27	28.85	1.0284
3	44	52.43	29.12	1.0561
5	44	48.32	31.48	1.0341
7	66	51.82	30.46	1.0743
10	45	54.62	28.83	1.0933
15	56	48.40	28.31	1.1088
20	46	53.86	32.44	1.1473
30	64	42.31	29.30	1.0779

Recovery of Market Value estimation

N = 96	Pre-Default	Default Date	RMV Estimates
Mean	48.4	48.6	1.0230 ??
Median	39	38.5	1.0013??
Standard Deviation	30.6	30.7	0.1824
First Quartile	21.5	22	0.9681
Third Quartile	67.55	69.375	1.0597

Economic default

- GJL(2008) proposed a statistical definition of the *economic default*;
- GJL (2008) showed that using the economic default date the average pricing error for distressed bond price is less than one basis point.


Time Between Economic Default Date and Announced Default Date
Cases
Days
0
5
10
15
20
25
30
35
0
20
40
60
80
100
120
140
160
180

Outline

How to model τ_r and τ_e?

Key: what is the economic reason for τ_e and τ_r?

Mathematical models of τ_r and τ_e

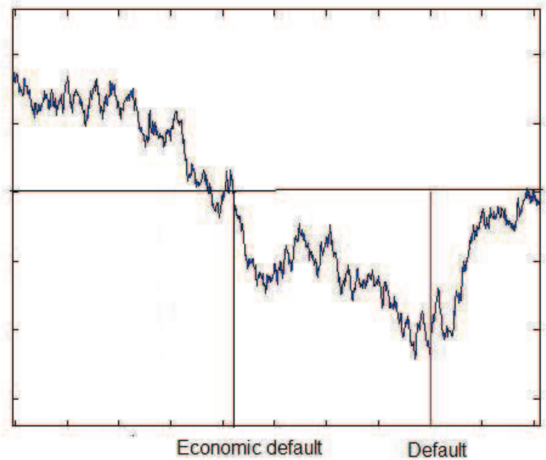
- Firm asset value process $S = (S_t, t \geq 0)$ is an exponential Lévy process;
- The firm needs to pay back its debt at *discrete times* $N_1, N_2, ..., N_n$ where $N_k = kN$ for some $N > 0$;
- *Real default* occurs when the asset value of the firm is less than the threshold D at a time Nk, i.e.

$$\tau_r = \inf\{Nk, S_{Nk} < D\}.$$

- *Economic default time* to be the *last time*, before the onset of recorded default, where the firm is able to make a debt repayment, i.e.

$$\tau_e = \sup_{\tau_r \geq t \geq \tau_r - N} \{t, S_t \geq D\}.$$

Default and economic default: illustration



Outline

Law of economic default τ_e

$$\tau_e = \sup_{\tau_r \geq t \geq \tau_r - N} \{t, S_t \geq D\}.$$

Technical difficulties:

- τ_e is not a stopping time.
- τ_e is a last passage time and not a first passage time.
- τ_e is defined conditioned on the knowledge of the real default τ_r.

Simple idea

- Consider $P_r(\tau_r - \tau_e | \tau_r)$
- The unconditioned law of $\tau_e - \tau_r$ is obtained by summing over all $n \geq 1$.

$\rightarrow$ For exponential of spectrally positive Lévy process, $\tau_r - \tau_e$ is a mixture of Arcsine law.

Example

Let $S = (S_t, t \geq 0)$ be a geometric Brownian motion so that $S_t = \exp(\mu W_t - \mu^2 t/2)$ under a risk neutral measure. Then, according to Yor et. al. (2008),

$$\mathbb{P}_{(u,D)}[\tau_r - \tau_e \in ds | \tau_r = N] = \frac{ds}{\pi\sqrt{s(N-u-s)}} \phi(\mu/2\sqrt{N-u-s}).$$

with $\phi(\mu) = \int_0^\infty dt e^{-t} \cosh(\mu\sqrt{2t})$.

Law of default τ_r

$$\tau_r = \inf\{Nk, S_{Nk} < D\}.$$

- τ_r is the first hitting time of a random walk $X = \{X_n = \log S_{nN}\}$ hitting level $\log D$;
- Analytical expression for τ_r using classical theory of random walk when debt level is constant or linear.

Analytical expression for τ_r

Given a random walk $\{\log S_{Nn}\}$ with an i.i.d sequence of $\{Y_n\}$ with a common distribution F and its associated green function U^+ and U^-. For any $y > 0, k \geq 1$,

$$P_r[\tau_r = kN, \log S_{\tau_r} \in dy] = \int_0^\infty \int_0^y \sum_{i=0}^{k-1} U^-(x - pN + y, k - i) U^+(dv - y, i) F(v - du).$$

Green function and computation

- Green functions of a random walk X is defined by its ladder time $T = \{T_n = (T_n^+, T_n^-)\}$ and ladder height process $H = \{H_n = (H_n^+, H_n^-)\}$

$$U^+(dx, i) = \sum_{n=0}^{\infty} P_r(H_i^+ \in dx,\ T_i^+ = n),$$

$$U^-(dx, i) = \sum_{n=0}^{\infty} P_r(H_i^- \in dx,\ T_i^- = n).$$

- From a computational perspective, Green functions can be derived by inverting appropriate Laplace transforms, such as from the well-known Friedst formula

$$1 - \mathbb{E}[r^{T_1^+} e^{itH_1^+}] = \exp(-\sum_{n=1}^{\infty} \frac{r^n}{n} \mathbb{E}[e^{it\log(S_{Nn})} : S_{nN} > 1]).$$

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Relation to existing approaches

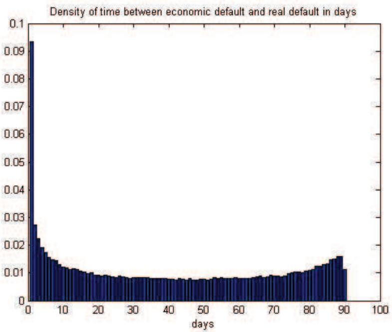
Simple application of Invariance Principle via appropriate scaling shows

- The classical structural model is a limiting case of our model as $N \to 0$.
- For better understanding of default, both discrete time and continuous time spaces are needed.
- Firm value process's parameter (either BM based or a more general Lévy model) can be estimated using the tail index and correlation structure of the firm's return.

  For example, the volatility coefficient of the firm value process σ_f is

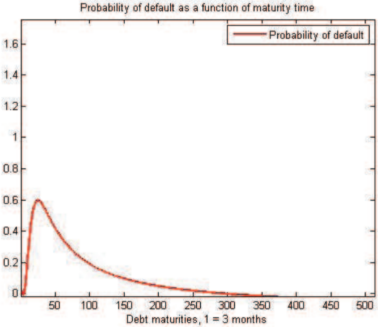
$$\sigma_f^2 = \frac{\mathbb{E}[Y_1^2] + 2\sum_{k=2}^{\infty} Cov(Y_1, Y_k)}{N}.$$

Example



The dynamics of the firm is a geometric Brownian motion, $\sigma = 0.25$, $r = 0.04$. Leverage $= 0.8$, $T = 3$ months, $B_1 = ... = B_8 = 0.4$.

Example



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Summary

- New concept and model of “economic default” is proposed and analyzed
- Quantitative analysis of default and economic default is consistent with empirical observation
- Many interesting problems follow, e.g.
    - pricing and hedging of credit derivatives under the new paradigm
    - probability questions involving Lévy processes, random walk, last passage time.

References

- X. Guo, H. Z. Lin, and R. Jarrow. Distressed debt prices and recovery rate estimation, *Review of Derivatives Research*, 2008.
- A. de Larrard, X. Guo, and R. Jarrow. The economic default time and the Arcsine Law (Available at www.ieor.berkeley.edu/ xinguo http://arxiv.org/abs/1012.0843.) 2010.

THANK YOU FOR YOUR ATTENTION