Analytic expansions of characteristic functions and densities, and applications in finance

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Abstract

Considerable classes of densities and characteristic functions are determined by partial integro-differential equations where initial data are Dirac delta-distributions and their Fourier transforms (analytic data), respectively.

In [Belomestny, D., Kampen, J., Schoenmakers, J.G.M.] it is observed that certain sets of analytic data form a class of analytic vectors for classes of processes with affine generators. This observation can be used in order to construct the characteristic function of a considerable class of affine processes from the operator symbols. Moreover, for this class characteristic functions can be locally represented by a functional series consisting of operator symbol functions and their formal derivatives, and with integer coefficients which can be explicitly computed (cf. [Kampen, J.(b)]). But it is difficult to generalize this (go beyond affine processes). We explain why.

For a certain class of local operators with nonlinear coefficients, i.e. classical strictly parabolic operators of second order with smooth bounded coefficients, related densities may be locally expanded in WKB-form. This may lead to efficient and accurate approximations. Indeed, in a realistic situation of a Libor market model environment we computed interest rate options with maturity of 10 years in one time step using only the first two correction terms of the Gaussian in the WKB-expansion of a related density (cf. [Kampen, J.; Kolodko, A., Schoenm., J.]). However, higher order approximations in WKB-form can be numerically unstable. In order to obtain stable higher order expansions we consider local expansions of the form

$$p(t,x,y) = \frac{1}{\sqrt{4\pi t}^{n}} \exp\left(-\frac{d_R^2(x,y)}{4t}\right)\left(\sum_{k=0}^{\infty} d_k t^k\right), \tag{1}$$

where d_R is Varadhan's leading term (a Riemannian metric with line-element determined by the inverse of the diffusion matrix) but -different to the WKB-expansion-the higher order corrections $d_k, k \geq 0$ are not part of the exponent (and more difficult to compute). In case of an Euclidean leading term and for a considerable class of non-linear drifts we can compute the d_k explicitly from recursive equations and determine a lower bound of the radius of convergence (cf. [Kampen, J.(c)]). We also discuss the extension to the general case with the analysis of the eikonal equation which determines the Riemannian metric d_R. While this metric exists only for strictly parabolic operators (at least in a regular sense), we finally discuss how density expansions of type (1) may be used for computation of specific semi-elliptic problems arising in applications such as Greeks for American options (cf. [Kampen, J.(a)]) or reduced Libor market models (cf. [Fries & Kampen (a), Fries C., Kampen, J. (b)].

References

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