A MULTIPLICATIVE APPROACH TO RANDOM TIMES

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Abstract

We first show, using the concept of a multiplicative system introduced in Meyer [6], that for any given positive $\mathbb{F}$-supermartingale G such that $G_\infty = 0$, there exists a random time τ on a suitable extension $(\widehat{\Omega}, \widehat{\mathcal{F}}, \widehat{\mathbb{F}}, \widehat{\mathbb{P}})$ of the underlying probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$ such that G is the Azéma supermartingale of τ or, more explicitly, $G_t = \widehat{\mathbb{P}}(\tau > t \,|\, \mathcal{F}_t)$ for every $t \in \mathbb{R}$. Moreover, the restriction of the probability measure $\widehat{\mathbb{P}}$ to the filtration $\mathbb{F}$ coincides with $\mathbb{P}$. This provides a solution to the problem of finding a random time with a given Azéma semimartingale. We then analyze properties of solutions to the abovementioned problem in terms of the so-called hypotheses (H) and (HP). Subsequently, we derive, under suitable technical assumptions, the semimartingale decomposition of an $\mathbb{F}$-local martingale with respect to the progressively enlarged filtration $\mathbb{G}$. We compare our result with previously established semimartingale decompositions for the special cases when τ is an honest time (see Jeulin and Yor [4]) or an initial time (see Jeanblanc and Le Cam [2]). Finally, the univariate construction is extended to the case of several correlated random times with predetermined Azéma semimartingales. Related results were obtained in a recent work by Jeanblanc and Song [3], who developed an approach different from ours and who worked under more restrictive conditions on G.

References

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