PROGRAM AND ABSTRACT

Program

<u>September 8, 2016</u>

- 10:00 - 11:30: Registration
- 13:30 - 14:20: Marcin Marciniak (University of Gdansk): Merging of positive maps: exposed and optimal maps, and their applications

Break

- 14:30 –15:20: Jun Ichi Fujii (Osaka Kyoiku Univ.): Introduction to TQC theory and spin networks

Break

- 15:40 - 16:30: Miklós Pálfia (Sungkyunkwan Univ. and Hungarian Academy of Sciences, Hungary): Loewner ' s theorem in several variables

Break

- 16:40– 17:30;Yongdo Lim (Sungkyunkwan Univ.): Ando-Hiai inequality for probability measures
- 18:00 Welcoming party

<u>September 9, 2016</u>

- 9:30 –10:20: Rajarama Bhat (ISI, Bangalore): Bures distance for completely positive maps

Break

- 10:30 –11:20: Wai Shing Tang (National Univ. of Singapore): All 2-positive linear maps from M_3 to M_3 are decomposable.

Break

- 11:30 – 12:20: Hoang Phi Dung (Posts and Telecommunications Institute of Technology, Hanoi, Vietnam): Some Lojasiewicz inequalities and beyond

Lunch

- 13:30 - 14:20: Seung-Hyeok Kye (Seoul National Univ.): Detecting various kinds of entanglement in multi-qubit systems

Break

- 14:30 –15:20:Benoit Collins (Kyoto Univ.): to be announced.

Break

- 15:30 - 16:00: Shigeru Furuichi (Nihon Univ.):On some inequalities for symmetric divergence measures

Break

- 16:00 – 16:30: Yoichi Udagawa (Tokyo University of Science): Parameterized operator means and operator monotonicity of expf(x)

Break

- 16:40 –17:30: Gen Kimura (Shibaura Institute of Technology): Information gain and storage in General Probabilistic Theories.
- 18:00 :Dinner party

September 10, 2016

- 9:30-10:20: Fumio Hiai (Tohoku Univ.): A concise survey of log-majorizations for matrices with applications to quantum information

Break

- 10:30-11:20: Le Cong Trinh (Quy Nhon Univ.): On the location of eigenvalues of matrix polynomials
- Closed ceremony

Abstract

- Marcin Marciniak (University of Gdansk)
  Title: Merging of positive maps: exposed and optimal maps, and their applications
  Abstract:

Let K_1, K_2, H_1, H_2 be Hilbert spaces and let $\phi_i : B(K_i) \to B(H_i)$, $i = 1, 2$, be positive linear maps. Consider Hilbert spaces $K = \bigoplus_{i=1}^3 K_i$ and $H = \bigoplus_{i=1}^3 H_i$, where $K_3 = H_3 = \mathbb{C}$. An element $X \in B(K)$ can be represented as a matrix $X = (X_{ij})_{i,j=1,2,3}$, where $X_{ij} \in B(K_j, K_i)$. In particular $X_{i3} \in B(\mathbb{C}, K_i) = K_i$, $X_{3j} \in B(K_j, \mathbb{C}) = K_j^*$, $i, j = 1, 2$, and $X_{33} \in \mathbb{C}$. We consider a map $\phi : B(K) \to B(H)$, which is of the form

$$\phi(X) = \begin{pmatrix} \phi_1(X_{11}) + \omega_2(X_{22})E_1 & 0 & B_1X_{13} + C_1X_{31}^t \\ 0 & \phi_2(X_{22}) + \omega_1(X_{11})E_2 & B_2X_{23} + C_2X_{32}^t \\ X_{31}B_1^* + X_{13}^tC_1^* & X_{32}B_2^* + X_{23}^tC_2^* & X_{33} \end{pmatrix},$$

where ω_i is a positive functional on $B(K_i)$, $B_i, C_i \in B(K_i, H_i)$, and $E_i \in B(H_i)$ is a projection onto the range of the operator $\phi_i(\mathbb{1}_{B(K_i)})$, $i = 1, 2$. The map ϕ is called a *merging of ϕ_1 and ϕ_2 by means of the ingredients ω_i, B_i, C_i*.

We examine properties of the above operation. In particular, we provide conditions on the ingredients ω_i, B_i, C_i which guarantee positivity of the map ϕ. It turns out that positivity of ϕ implies $\phi_i + \chi_i \leq \phi_i$, $i = 1, 2$, where $\phi_i, \chi_i : B(K_i) \to B(H_i)$ are defined by

$$\phi_i(X) = B_iXB_i^*, \qquad \chi_i(X) = C_iX^tC_i^*, \qquad X \in B(K_i).$$

Further, using results of [1] we show that for any pair of maps ϕ_1, ϕ_2 such that ϕ_1 is 2-positive and ϕ_2 is 2-copositive, there are ingredients such that the merging by means of them is a nondecomposable map.

Next, we present some examples. The first one is the map $\phi : \mathbb{M}_3(\mathbb{C}) \to \mathbb{M}_3(\mathbb{C})$

$$\phi\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(x_{11} + x_{22}) & 0 & \frac{1}{\sqrt{2}}x_{13} \\ 0 & \frac{1}{2}(x_{11} + x_{22}) & \frac{1}{\sqrt{2}}x_{32} \\ \frac{1}{\sqrt{2}}x_{31} & \frac{1}{\sqrt{2}}x_{23} & x_{33} \end{pmatrix}$$

described by Miller and Olkiewicz [4].

We provide a high dimensional generalization of the above map. Namely, we consider merging of maps

$$\phi_1(X) = A_1XA_1^*, \quad X \in B(K_1), \qquad \phi_2(X) = A_2XA_2^*, \quad X \in B(K_2),$$

for some $A_i \in B(K_i, H_i)$, $i = 1, 2$. It was shown in [2] that these maps are exposed. The suitable ingredients are

$$B_1 = A_1, \quad B_2 = 0, \quad C_1 = 0, \quad C_2 = A_2, \quad \omega_1(X) = \mathrm{Tr}(A_1XA_1^*), \quad \omega_2(X) = \mathrm{Tr}(A_2X^tA_2^*).$$

It turns out that this generalization provides an example of exposed positive map.

We also describe a generalization of the optimal map described in [5] and show that it is also an optimal map.

Next, we consider a class of maps acting on $\mathbb{M}_3$ into itself which are of the form

$$\phi\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix} = \begin{pmatrix} f_1x_{11} + w_2x_{22} & 0 & b_1x_{13} + c_1x_{31} \\ 0 & f_2x_{22} + w_1x_{11} & b_2x_{23} + c_2x_{32} \\ \overline{b_1}x_{31} + \overline{c_1}x_{13} & \overline{b_2}x_{32} + \overline{c_2}x_{23} & x_{33} \end{pmatrix}$$

We characterize all maps among them which are positive. We provide also conditions for extremality as well as for optimality and nondecomposability.

Finally, we discuss some applications of the maps described above to detection of entanglement of states.

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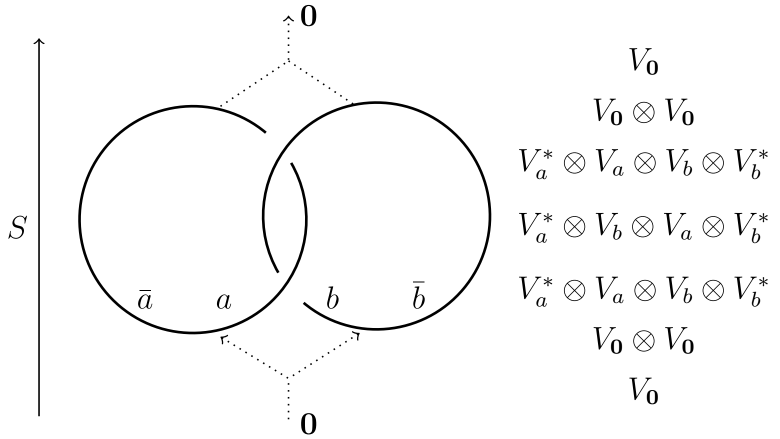
- Jun Ichi Fujii (Osaka Kyoiku Univ.)
  Title: Introduction to TQC theory and spin networks
  Abstract:

The TQC theory was known mainly by the issue in the Scientific American 2006 April. The term 'TQC' means the **topological quantum computation**, which is one of remarkable theories supported by the Microsoft to realize quantum computers. This theory is related to various field; theory of quantum groups, Lie algebras, Hopf algebras and conformal field or string theory. It is discussed basically in a modular tensor category where the modularity is the invertibility of S matrix expressed by the following Hopf link:



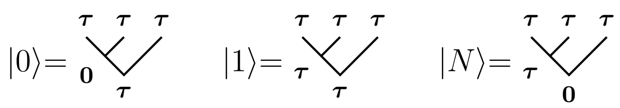
The leading concept is 'anyon' with its fusion or splitting. In the quantum particles, the Boson or the Fermion is discussed in the symmetric or anti-symmetric tensor products; $|y\rangle \otimes |x\rangle \sim |x\rangle \otimes |y\rangle$ or $|y\rangle \otimes |x\rangle \sim -|x\rangle \otimes |y\rangle$ respectively. Recently in the special situations, other types of particles (quasi-particles) like Majorana Fermions are considered: **abelian anyon**; $|y\rangle \otimes |x\rangle \sim e^{it}|x\rangle \otimes |y\rangle$ or **non-abelian anyon**; $|y\rangle \otimes |x\rangle \sim e^{iH}(|x\rangle \otimes |y\rangle)$. TQC theory is based on non-abelian anyons. In this talk, we introduce this theory and its simple model 'the spin networks'.

Let $\mathcal{F}$ be a finite set of (quasi-)particles closed in fusion actions. We assume that it is *dual* ($a \in \mathcal{F} \Rightarrow \bar{a} \in \mathcal{F}$, where $\bar{a}$ is anti-particle of a) and contains the *vacuum* one $\mathbf{0}$. As for the fusion rule, for the non-negative integer N_{ab}^c for $a, b, c \in \mathcal{F}$,

$$a \otimes b \to \sum_{x \in \mathcal{F}} N_{ab}^x x, \quad N_{ab}^c = N_{ba}^c, \quad N_{a\mathbf{0}}^b = \delta_{a,b}$$

where the sum means all possibility of changing of $a \otimes b$.

Fibonacci anyon gives a simple example: $\mathcal{F} = \{\mathbf{0}, \tau = \bar{\tau}\}$, $\tau = \mathbf{0} + \tau$. Thus τ is non-abelian anyon by $\sum_x N_{\tau\tau}^x = 2 > 1$. Reflecting this difference of states, the fusion trees (corresponding to conformal blocks) define the base state vectors:

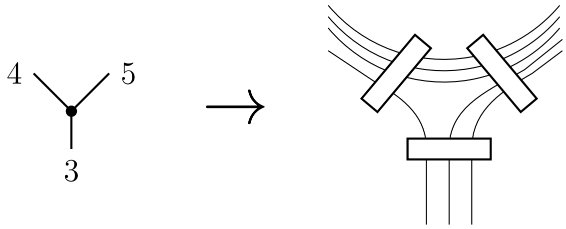


The third one is essentially the vacuum and then is the identity element, so that we can neglect it. The particle braiding (R matrix) yields the braiding of vectors, which is called B matrices. There is a non-diagonal B matrix which is why it is called 'non-abelian'. Another important matrix is F; particle flipping (or recoupling). These matrices are obtained by the categorical rules (MacLane coherence theorem); F matrix is obtained by the pentagon rule and R matrix is by the hexagon one. Based on these, B matrices can construct quantum gates approximately, which is the topological quantum computing.

TQC theory has a skein expression. A typical difference between the knot theory and TQC is the toplogical spin. In the knot theory, it is nothing but a straight line via the Reidemeiater move, but in TQC, the nontrivial phase θ appears. Thus the TQC theory fits the bracket polynomials rather than the Jones ones.

Other outstanding property of TQC is the ribbon expression. The modular tensor category in TQC is also a ribbon category, so that some formulae are easily obtained by this expression.

In the last half of this talk, we introduce spin networks based on the Jones-Wenzl projections and the Temperly-Lieb algebra, which is one of the simple model of TQC. The difference of particle is based on the question 'How many lines does each line in the skein diagram consist of?': The admissible 3-valent graph has the following expression (where the box stands the Jones-Wenzl projection):



The evaluations for θ-net and the tetrahedron net are important in the spin networks. These evaluations are complicated but they yield those of 6-j symbols which is equivalent to F matrix. By the transformation from spin networks to TQC, we also obtain an F matrix formula in TQC.

• Miklós Pálfia (Sungkyunkwan Univ. and Hungarian Academy of Sciences, Hungary)
Title: LOEWNER'S THEOREM IN SEVERAL VARIABLES

Abstract:

We provide characterizations of operator monotone and concave func- tions in several operator variables using LMIs and the theory of matrix convex sets. This completes the work of Agler-McCarthy-Young [2] providing characterizations restricted for commutative tuples of oper- ators, hence to the several real variable situation, the work of Helton-McCullough-Vinnikov [6] characterizing free rational - thus already analytic - several variable matrix convex functions and the work of Pascoe-Tully-Doyle [15] characterizing free analytic matrix monotone functions in several variables.

For a free operator concave function we define its hypograph as the downward saturation of its graph with respect to the positive definite order. Then operator concavity of a free function is characterized by the matrix convexity of its hypograph. Given a closed matrix con- vex hypograph as a subset of a Cartesian product of the linear space of bounded linear operators, one can find its supporting linear functionals and represent them as linear pencils of operators on the tensor product of the linear space with its dual space. Then the linear pencil defines a linear matrix inequality (LMI) such that its extremal solution coin- cides with the value of the operator concave function. We establish an explicit solution formula for the extremal solutions of this LMI using the Schur complement. This LMI solution technique alone seems to have further applications to the general theory, in particular analytic rigidity, of matrix convex sets and LMIs.

The above approach leads to the extension of Loewner's classical representation theorem of operator concave and operator monotone functions from 1934, into the non-commutative several variable situa- tion. Our theorem states that a free function defined on a k-variable free self-adjoint domain is operator monotone if and only if it has a free analytic continuation to the upper operator poly-halfspace $Pi^k := \{X2B(E)^k : \mathcal{F}Xi > 0, 1 \leq i \leq k\}$ for any Hilbert space E, mapping Π^k to Π. This approach also provides a new proof to the one-variable case of Loewner's theorem.

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cance of Loewner's theorem

- Yongdo Lim (Sungkyunkwan Univ.)
  Title: Ando-Hiai inequality for probability measures
  Abstract:

We establish an order inequality of probability measures on partially ordered symmetric spaces of non-compact type, namely symmetric cones (self-dual homogeneous cones), characterizing the Cartan barycenter among other invariant and contractive barycenters. The derived inequality and partially ordered structures on the probability measure space lead also to significant results on inequalities including the Ando-Hiai inequality for probability measures on symmetric cones.

- Rajarama Bhat (Indian Statistical Institute, Bangalore)
  Title: Bures distance for completely positive maps
  Abstract:

  D. Bures had defined a metric on the set of normal states on a von Neumann algebra using GNS representations of states. This notion has been extended to completely positive maps between C*-algebras by D. Kretschmann, D. Schlingemann and R. F. Werner. They also explored applications of the notion in quantum information. We present a Hilbert C*-module version of this theory. We show that we do get a metric when the completely positive maps under consideration map to a von Neumann algebra. Further, we include several examples and counter examples. We also prove a rigidity theorem, showing that representation modules of completely positive maps which are close to the identity map contain a copy of the original algebra. This is a joint work with K. Sumesh.

- Wai Shing Tang (National Univ. of Singapore)

  Title: All 2-positive linear maps from M_3 to M_3 are decomposable.

  Abstract:

  Following an idea of Choi, we obtain a decomposition theorem for k-positive linear maps from M_m to M_n, where $2 \leq k < \min\{m, n\}$.

  As a consequence, we give an affirmative answer to Kye's conjecture (also solved independently by Choi) that every 2-positive linear map from M_3 to M_3 is decomposable.

  This is joint work with Yu Yang and Denny H. Leung.

• Hoang Phi Dung (Posts and Telecommunications Institute of Technology, Hanoi, Vietnam)
Title: Some Lojasiewicz inequalities and beyond
Abstract:

In this talk, we give some Łojasiewicz-type inequalities for continuous definable functions in an o-minimal structure. We also give a necessary and sufficient condition for which global error bound exists and the relationship between the Palais-Smale condition and this global error bound. In beyond, we study some facts related to matrices.

Some Łojasiewicz inequalities

Let $f:\mathbb{R}^n \to \mathbb{R}$ be a real analytic function with $f(0)=0$. Let $V := \{x \in \mathbb{R}^n | f(x) = 0\}$ and K be a compact subset in $\mathbb{R}^n$. Then the (classical) Łojasiewicz inequality (see [1]) asserts that:

− There exist $c > 0, \alpha > 0$ such that

$$|f(x)| \geq c d(x,V)^{\alpha} \quad \text{for } x \in K. \tag{1}$$

Let $f:\mathbb{R}^n \to \mathbb{R}$ be a real analytic function with $f(0)=0$ and $\nabla f(0) = 0$. The Łojasiewicz gradient inequality (see [1]) asserts that:

− There exist $C > 0, \rho \in [0,1)$ and a neighbourhood U of 0 such that

$$\|\nabla f(x)\| \geq C|f(x)|^{\rho} \quad \text{for } x \in U. \tag{2}$$

We consider these inequalities in the case of o-minimal structures. Roughly speaking, o-minimal structures are systems of subsets of $\mathbb{R}^k, k = 1, 2, \ldots$ and functions on these subsets. The functions contain polynomials and have geometric properties which are analogous to polynomials.

Theorem (H. [6]). Let $f\colon \mathbb{R}^n \to \mathbb{R}$ be a continuous definable function. Assume that $S := \{x \in \mathbb{R}^n \mid f(x) \leq 0\} \neq \emptyset$ and $[f(x)]_+ := \max\{f(x), 0\}$. Then the following two statements are equivalent

(i) For any sequence $x^k \in \mathbb{R}^n \setminus S, x^k \to \infty$, we have
   (i1) if $f(x^k) \to 0$ then $d(x^k, S) \to 0$;
   (i2) if $d(x^k, S) \to \infty$ then $f(x^k) \to \infty$.

(ii) There exists a function $\mu : [0, +\infty) \to \mathbb{R}$, which is definable, strictly increasing and continuous on $[0, +\infty)$ with $\mu(0) = 0$, $\lim\limits_{t\to+\infty} \mu(t) = +\infty$, such that

$$d(x, S) \leq \mu([f(x)]_+), \quad \forall x \in \mathbb{R}^n.$$

Theorem (H. [6]). Let $f : \mathbb{R}^n \to \mathbb{R}$ be a continuous definable function in some o-minimal structure and suppose that $\widetilde{K}(f) \cap (-\epsilon, \epsilon) = \{0\}$. Then the following two statements are equivalent.

(i) For any sequence $x^k \to \infty$, $\mathfrak{m}_f(x^k) \to 0$ implies $f(x^k) \to 0$.

(ii) There exists a function $\varphi : (0, \delta) \to \mathbb{R}$, which is definable, monotone and continuous such that

$$\mathfrak{m}_f(x) \geq \varphi(|f(x)|), \quad \forall x \in f^{-1}(D_\delta).$$

where $\mathfrak{m}_f(x)$ is the limiting subdifferential of f.

Let $\nu(A)$ is the minimum of the eigenvalues of $\sqrt{AA^*}$.

Proposition (D. - Toan HM). Let $A(x)$ be a real matrix polynomial (entries $a_{ij}(x)$ are the real polynomials). Then, for $x \sim 0$, there exist positive numbers c, l such that

$$\nu(A(x)) \geq c.dist(x, \{\nu(A(x)) = 0\})^l.$$

Acknowledgments. We would like to thank Prof. Ha Huy Vui for his proposing the problem, Assoc. Prof. Pham Tien Son for his suggestions and Dr. Dinh Si Tiep for his useful discussions.

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- Seung-Hyeok Kye (Seoul National Univ.)
  Title: Detecting various kinds of entanglement in multi-qubit systems
  Abstract:

We interpret multi-qubit entanglement witnesses as various kinds of positivity of multi-linear maps and/or linear maps. We apply our results to characterize several kinds of separability of X-shaped states including Greenberger-Horne-Zeilinger diagonal states.

- Benoit Collins (Kyoto Univ.)
  Title: Positive maps from free probability theory
  Abstract:

We interpret the super convergence properties of the free additive convolution semigroup in terms of Choi matrices, and provide new examples of maps that are positive but not completely positive. In particular, we show that some of our constructions yield new examples of indecomposable positive maps and describe some applications to the geometry of the set of separable states. This is joint work with Patrick Hayden and Ion Nechita.

- Shigeru Furuichi (Nihon Univ.)

Title: On some inequalities for symmetric divergence measures

Abstract:

In the paper [1], the tight bounds for symmetric divergence measures are derived by applying the results established in the paper [2]. In the paper [1], the minimization problem for Bhattacharyya coefficient, Chernoff information, Jensen-Shannon divergence and Jeffrey's divergence under the constraint on total variation distance. In this article, we are going to report two kinds of extensions for the above results, mainly classical q-extension and partially noncommutative (quantum) extension. The parametric q-extension means that Tsallis entropy $H_q(X) \equiv -\sum_x p(x)^q \ln_q p(x)$ [3] converges to Shannon entropy when $q \to 1$. Namely, all results with the parameter q recover the usual (standard) Shannon's results when $q \to 1$. However the non-additivity $\ln_q(xy) = \ln_q y + (1-q)\ln_q x \ln_q y$ of q-logarithmic function defined by $\ln_q(x) \equiv \dfrac{x^{1-q}-1}{1-q}$, $(q \neq 1, x \geq 0)$ often disable some computations, while the case $q = 1$ goes well. As you know, non-commutative case also gives us difficulties for the computations due to its non-commutativity.

In my talk, I am going to talk about the lower bound for Jensen-Shannon-Tsallis diverence with parameter q is given by applying the results in [2]. In addition, the lower bound for Jeffrey- Tsallis divergence with parameter q is also given by applying the results in [2] and deriving q-Pinsker's inequality for $q \geq 1$. This implies new upper bounds of $\sum_{u\in\mathcal{U}} |p(u) - Q_{d,l}(u)|$ for general probability distribution $p(u)$ and the probability distribution $Q_{d,l}(u)$ composed by the elements of the terms of the left hand side of Kraft inequality, with the q parametric extended average code length $\overline{n}q$. (However I am not likely this nq since its form is slightly complicated.) Finally, the lower bound for quantum Jeffrey divergence is given by applying the monotonicity (data processing inequality) of quantum f-divergence.

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• Yoichi Udagawa (Tokyo University of Science)

Title: Parameterized operator means and operator monotonicity of $\exp f(x)$.

Abstract:

A map $\mathcal{M}$ is called an operator mean if the operator $\mathcal{M}(A;B)$ satis
es the following four conditions; for positive operators $A, B, C, D \geq 0$, 1. $A \leq C$ and $B \leq D$ imply $\mathcal{M}(A;B) \leq \mathcal{M}(C;D)$, 2. $X(\mathcal{M}(A;B))X \leq \mathcal{M}(XAX;XBX)$ for all self-adjoint operator X, 3. $A_n \searrow A$ and $B_n \searrow B$ imply $\mathcal{M}(A_n;B_n) \searrow \mathcal{M}(A;B)$ in the strong topology, 4. $\mathcal{M}(I;I) = I$.

Each operator mean is often identi
ed with its representing function, namely corresponding operator monotone function, by Kubo-Ando theory.

Recently, we have constructed a 2-parameter family of operator monotone function $\{F_{r,s}(x)\}_{r,s\in[-1,1]}$ by integrating the function $P_{s,\alpha}(x)$ of the parameter $\alpha \in [0,1]$ ([2]);

$$F_{r,s}(x) = \left(\frac{r(x^{r+s}-1)}{(r+s)((x^s-1)} \right)^{\frac{1}{s}} \quad (x > 0)$$

This family interpolates many famous operator monotone functions and has monotonicity for its parameters $r, s \in [-1,1]$. In this talk, we treat this family as $S_{p,\alpha}(x) = \left(\frac{p(x^\alpha-1)}{\alpha(x^p-1)}\right)^{\frac{1}{\alpha-p}}$. In [1], Nagisa and Wada have obtained an equavalent condition of parameters p and α such that $S_{p,\alpha}(x)$ is operator monotone. However, their characterization have not given any concrete range of parameters. Firstly, we extend the range of parameters p and α such that $S_{p,\alpha}(x)$ is operator monotone as follows;

Theorem 1.*Let*

$$S_{p,\alpha}(x) = \left(\frac{p(x^\alpha-1)}{\alpha(x^p-1)} \right)^{\frac{1}{\alpha-p}} \quad (x > 0).$$

Then $S_{p,\alpha}(x)$ *is operator monotone if* $(p,\alpha) \in \mathcal{A} \subset \mathbf{R}^2$*, where*

$$\mathcal{A} = ([-2,1] \times [-1,2]) \cup ([-1,2] \times [-2,1]) \cup \{(p,\alpha) \in \mathbf{R}^2 \mid \alpha = -p\}.$$

From the above theorem, we regard $S_{p,\alpha}(x)$ as the representing function of 2-parameter Stolarsky mean.

On the other hand, it is well known that $\exp(x)$ is not an operator mono- tone function, in contract with its inverse function log x is so. But there exists a function f(x) such that expff(x)g is an operator monotone function besides constant, like

$$\exp\left(\frac{x\log x}{x-1} - 1\right).$$

Secondly, we give a characterization of such function;

Theorem 2. *Let* $f(x)$ *be a continuous function on* $(0,\infty)$. *If f(x) is not a constant or* $\log(\alpha x)(\alpha > 0)$*, then the following are equivalent:*

(1) $\exp\{f(x)\}$ *is an operator monotone function,*

(2) $f(x)$ *is an operator monotone function, and there exists an analytic con- tinuation satisfying*

$$0 < \nu(r,\theta) < \theta,$$

where

$$f(re^{i\theta}) = u(r,\theta) + i\nu(r,\theta)(0 < r, 0 < \theta < \pi).$$

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- Gen Kimura (Shibaura Institute of Technology)

  Title: Information gain and storage in General Probabilistic Theories

  Abstract:

  We discuss upper bounds on both information gain and storage in general probabilistic theories (GPTs). Firstly, we introduce a systematic way to construct infinitely many entropies in GPTs, and as one of its application, we prove that Holevo theorem, which gives a famous upper bound of quantum accessible information can be generalized to hold in any GPT. Secondly, we show a general bound on information storage, and point out an interesting relation between the bound and the geometry of a state space.

- Fumio Hiai (Tohoku Univ.)

Title: A concise survey of log-majorizations for matrices with applications

Abstract:

This is a tutorial talk about how useful are log-majorizations for matrices to obtain matrix trace/norm inequalities. For two positive semidefinite $n \times n$ matrices A, B, the *weak-majorization* $A \prec_w B$ and the *supermajorization* $A \prec^w B$ are defined as

$$\sum_{i=1}^{k} \lambda_i(A) \le \sum_{i=1}^{k} \lambda_i(B), \qquad 1 \le k \le n,$$

$$\sum_{i=1}^{k} \lambda_{n+1-i}(A) \ge \sum_{i=1}^{k} \lambda_{n+1-i}(B), \qquad 1 \le k \le n,$$

respectively, where we denote $\lambda_1(A) \ge \cdots \ge \lambda_n(A)$ for the eigenvalues of A in decreasing order with counting multiplicities. On the other hand, the *weak log-majorization* $A \prec_{w \log} B$ and the *log-supermajorization* $A \prec^{w \log} B$ are defined as

$$\prod_{i=1}^{k} \lambda_i(A) \le \prod_{i=1}^{k} \lambda_i(B), \qquad 1 \le k \le n,$$

$$\prod_{i=1}^{k} \lambda_{n+1-i}(A) \ge \prod_{i=1}^{k} \lambda_{n+1-i}(B), \qquad 1 \le k \le n.$$

As is well-known, the notion of weak (log-)majorization is closely related to inequalities for symmetric (or unitarily invariant) norms. For instance, $A \prec_{w \log} B$ is equivalent to each of the following:

- $\|A^p\|_{(k)} \le \|B^p\|_{(k)}$ for all $p > 0$ and $1 \le k \le n$, where $\|\cdot\|_{(k)}$ denotes the Ky Fan k-norm,
- $\|f(A)\| \le \|f(B)\|$ for every continuous non-decreasing function $f : [0, \infty) \to [0, \infty)$ such that $f(e^x)$ is convex on $(-\infty, \infty)$, and for every unitarily invariant norm $\|\cdot\|$.

But in my talk I want to stress also that the notion of (log-)supermajorization is very related to symmetric anti-norms recently developed by Bourin and myself, which is not yet well written in the literature.

Next, I will survey Araki's log-majorization

$$(A^{q/2}B^qA^{q/2})^{1/q} \prec_{\log} (A^{p/2}B^pA^{p/2})^{1/p}, \qquad 0 < q < p,$$

and the log-majorization of Ando and myself

$$(A^p \#_\alpha B^p)^{1/p} \prec_{\log} (A^q \#_\alpha B^q)^{1/q}, \qquad 0 < q < p$$

in connection with the Furuta inequality, where $\#_\alpha$ denotes the weighted geometric mean with $0 \le \alpha \le 1$. Finally, I will exemplify the usefulness of log-majorization method in applications to inequalities between different types of quantum Rényi divergences.

- Le Cong Trinh (Quy Nhon Univ.)

Title: On the location of eigenvalues of matrix polynomials

Abstract.

Let $\mathbf{C}^{n\times n}$ denote the set of all $n\times n$ matrices whose entries in $\mathbf{C}$. For a *matrix polynomial* we mean the matrix-valued function in one complex variable of the form

$$P(z) = A_m z^m + \cdots + A_1 z + A_0,$$

where $A_i \in \mathbf{C}^{n\times n}$ for all $i = 0, \cdots, m$. If $A_m \neq 0$, $P(z)$ is called a matrix polynomial of *degree* m. When $A_m = I$, the identity matrix, the matrix polynomial $P(z)$ is called a *monic*.

A number $\lambda \in \mathbf{C}$ is called an *eigenvalue* of the matrix polynomial $P(z)$, if there exists a nonzero vector $x \in \mathbf{C}^n$ such that $P(\lambda)x = 0$. Then the vector x is called, as usual, an *eigenvector* associated to the eigenvalue λ. Note that each finite eigenvalue of $P(z)$ is a zero of the the *characteristic polynomial* $\det(P(z))$.

The *polynomial eigenvalue problem* (PEP) is to find an eigenvalue λ and a non-zero vector $x \in \mathbf{C}^n$ such that $P(\lambda) = 0$. For $m = 1$, (PEP) is actually the *generalized eigenvalue problem* (GEP)

$$Ax = \lambda Bx,$$

and, in addition, if $A_1 = I$, we have the standard eigenvalue problem

$$Ax = \lambda x.$$

For $m = 2$ we have the *quadratic eigenvalue problem* (QEP).

(QEP), and more generally (PEP), plays an important role in applications to science and engineering. We refer to [9] for a survey on applications of (QEP). Moreover, we refer to the book of I. Gohberg, P. Lancaster and L. Rodman [4] for a theory of matrix polynomials.

Computing eigenvalues of matrix polynomials (even computing zeros of univariate polynomials and eigenvalues of scalar matrices) is a hard problem. Therefore, it is useful to find the location of these eigenvalues. Note that, if A_m is singular, then $P(z)$ has an infinite eigenvalue, and if A_0 is singular then 0 is an eigenvalue of $P(z)$. Therefore, in order to find an upper bound and a lower bound for $|\lambda|$, *we always assume A_m and A_0 to be non-singular.*

In [5], N.J. Higham and F. Tisseur have given some bounds for eigenvalues of matrix polynomials based on the norm of their coefficient matrices. Continuing the idea of N.J. Higham and F. Tisseur, in this talk we establish some other bounds for the module of eigenvalues of the matrix polynomial $P(z)$, generalize some known results on the location of zeros of univariate polynomials given in [1, 2, 3, 6, 7, 8], and compare these bounds to those given by N.J. Higham and F. Tisseur.

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