Some properties of weighted operator means

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Let $\mathfrak{M}(\cdot,\cdot)$ be an operator mean with the representing function $f(x)$. If the condition $f'(1) = t$ is satisfied, then $\mathfrak{M}(\cdot,\cdot)$ is said to be a t-weighted operator mean. For example, the operator mean with the representing function

$$P_{r,t}(x) := [(1-t) + tx^r]^{\frac{1}{r}} \ (r \in [-1,1])$$

is a t-weighted operator mean for $t \in [0,1]$. This mean is called the weighted power mean and denoted by $\mathfrak{P}_{r,t}(\cdot,\cdot)$. Pálfia and Petz [2] recently suggested an algorithm for making t-weighted operator means $\{\mathfrak{M}_t(\cdot,\cdot)\}$ from a given operator mean $\mathfrak{M}(\cdot,\cdot)$ and a constant $t \in [0,1]$. Its procedure can be regarded as a kind of binary search algorithm. We show some properties about $\{\mathfrak{M}_t(\cdot,\cdot)\}$ obtained by this algorithm.

On the other hand, J. I. Fujii and E. Kamei defined "operator interpolational means" and recently J. I. Fujii introduced their characterization [1]. The operator interpolational mean is a family of weighted operator means which satisfies the following "interpolational condition"

$$\mathfrak{M}_\delta(\mathfrak{M}_\alpha(A,B), \mathfrak{M}_\beta(A,B)) = \mathfrak{M}_{(1-\delta)\alpha+\delta\beta}(A,B)$$

for $\alpha, \beta, \delta \in [0,1]$ and $A, B > O$. We give a characterization of operator interpolational means, and this characterization gives us the fact that the weighted power mean $\mathfrak{P}_{r,\alpha}(\cdot,\cdot)$ is "the only" operator interpolational mean.

References

[1] J. I. Fujii, *Interpolationality for symmetric operator means*, Sci. Math. Jpn., **75** (2012), 267–274.

[2] M. Pálfia and D. Petz, *Weighted multivariable operator means of positive definite operators*, Linear Algebra Appl., **463** (2014), 134–153.