Semitoric systems in geometry and dynamics

by

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Integrable systems are systems of differential equations with 'many symmetries'. For instance, spinning tops, the spherical pendulum, coupled angular momenta, but also soliton waves in shallow water fall in this category. In addition, it is often more accessible to model an originally nonsymmetric phenomenon by assuming a number of symmetries.

Depending on the setting, aim, and aspects, there are various notions of integrability such as Frobenius integrability, Liouville integrability, algebraic integrability, integrable hierarchies, integrable lattices etc.

In this talk, we are interested in a special class of Liouville integrable Hamiltonian systems on 4-dimensional manifolds, namely so-called *semitoric systems*. Roughly speaking, a semitoric system is a completely integrable Hamiltonian system on a 4-dimensional manifold that admits only nondegenerate singularities without hyperbolic components and whose flow gives rise to an $(\mathbb{S}^1 \times \mathbb{R})$-action. Semitoric systems have been symplectically classified a few years ago by Pelayo & Vu Ngoc by means of five invariants. Recently, there has been made considerable progress by various authors concerning the computation and geometric implications of these invariants.

In this talk, we will motivate the importance and usefulness of semitoric systems, explain their singularities and possible types of fibers in a geometric way and illustrate it with accessible examples, and give an overview of the progress during the last years.