Fragmentation Process derived from α-stable Galton-Watson trees

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Abstract

Aldous, Evans and Pitman (1998) studied the behaviour of the fragmentation process derived from deleting the edges of a uniform random tree on n labelled vertices. In particular, they showed that, after proper rescaling, the above fragmentation process converges as $n \to \infty$ to the fragmentation process of the Brownian CRT obtained by cutting down the Brownian CRT along its skeleton in a Poisson manner.

In this series of talks, we will discuss the fragmentation process obtained by deleting randomly chosen edges from a critical Galton-Watson tree $\mathbf{t}_n$ conditioned on having n vertices, whose offspring distribution belongs to the domain of attraction of a stable law of index $\alpha \in (1, 2]$. The main result establishes that, after rescaling, the fragmentation process of $\mathbf{t}_n$ converges, as $n \to \infty$, to the fragmentation process obtained by cutting down proportional to the length of the skeleton of an α-stable Lévy tree. We will also explain how one can construct the latter by considering the partitions of the unit interval induced by the normalized α-stable Lévy excursion with a deterministic drift. In particular, the above extends the result of Bertoin (2000) on the fragmentation process of the Brownian CRT.

The approach uses the well-known Prim's algorithm (or Prim-Jarník algorithm) to define a consistent exploration process that encodes the fragmentation process of $\mathbf{t}_n$. We will discuss the key ideas of the proof.

Joint work with Cecilia Holmgren (Uppsala University).

The plan:

Lecture 1: (Setting the Stage)Introduces key concepts and definitions (Random trees, Galton-Watson trees, Brownian CRT, Stable Lévy Tree). Presents the main result.

Lecture 2: Briefly reviews the key points from Lecture 1 to refresh everyone's memory. Discusses connections to other models (Additive coalescents, Laminations, Trees with given degrees, Inhomogeneous CRT). Introduces the main ideas behind the proof of the main result.