13種のC60フラーレンと
離散ソボレフ不等式の最良定数

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武村 一雄（日大理工）

The best constant of discrete Sobolev inequality on 13 kinds of the C60 Fullerene

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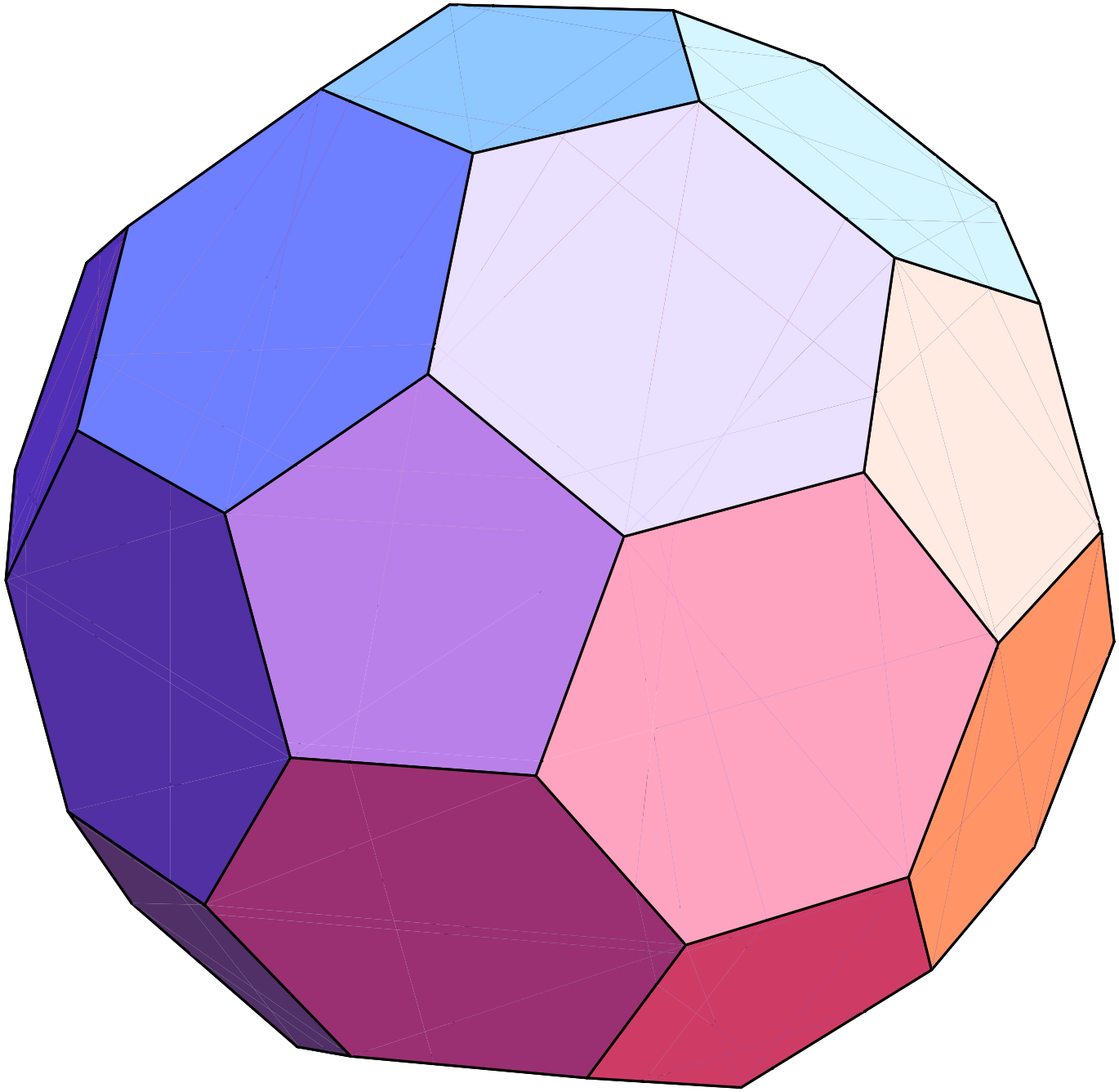
Atsushi Nagai (Nihon Univ.)

Hiroyuki Yamagishi
(Tokyo Metropolitan College)

Kazuo Takemura (Nihon Univ.)

C60 Fullerene



C60フラーレン

炭素原子60，５員環12，６員環20

代表はバッキーボール，切頂正20面体

1985年　ハロルド・クロトー，
リチャード・スモーリー，ロバート・カールが発見

1996年　ノーベル化学賞

1970年　大澤映二がバッキーボールの存在を予言

カーボンナノチューブ

1991年　飯島澄男が発見

ジグザグ型，アームチェア型は良導体

カイラル型は半導体

古典力学モデル

辺の両端の二つの炭素原子は線形バネで結ばれる

バネ定数一様

バッキーボールは非常に固い

固さは離散ソボレフ不等式の最良定数

対称性

バッキーボールは切頂正２０面体

平行でない３本のジグザグリングをもつもの

平行でない２本のジグザグリングをもつもの

それらを回転したもの計１３種

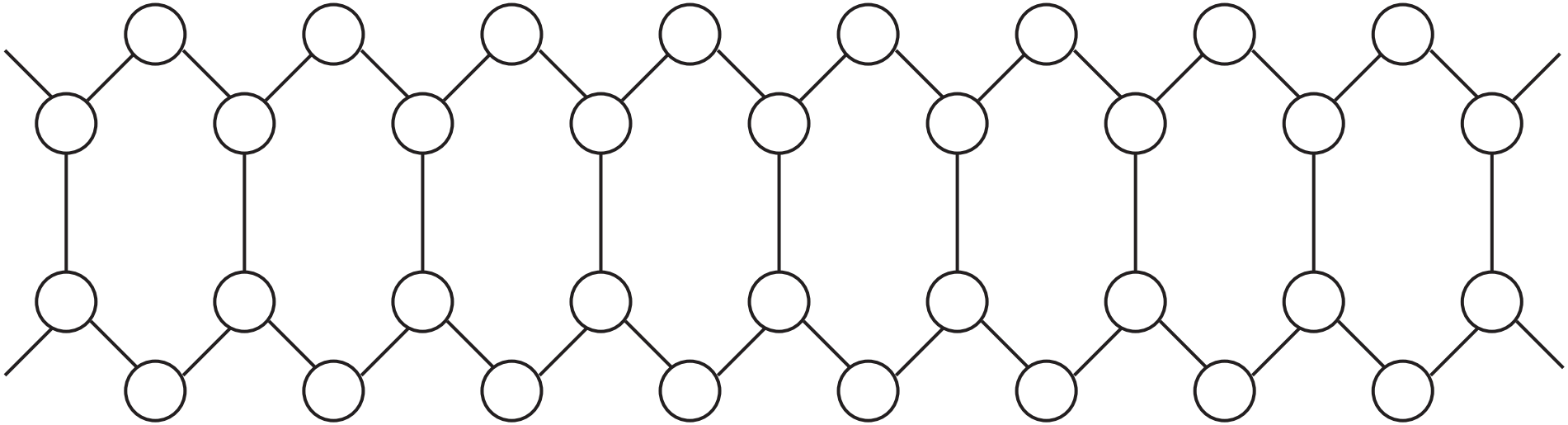
１３種Ｃ60

２種の離散ソボレフ不等式の最良定数を求めた

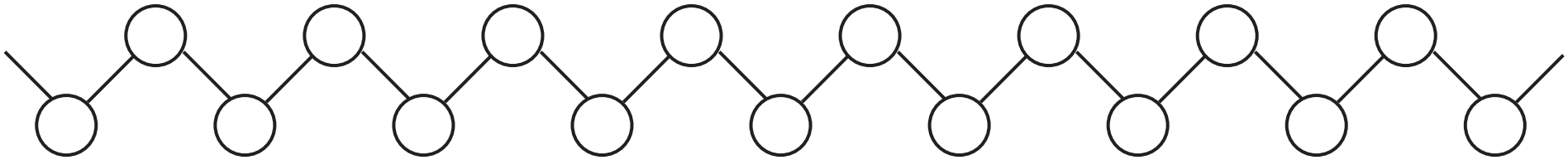
バッキーボールの最良定数が一番小さい

すなわち一番固い

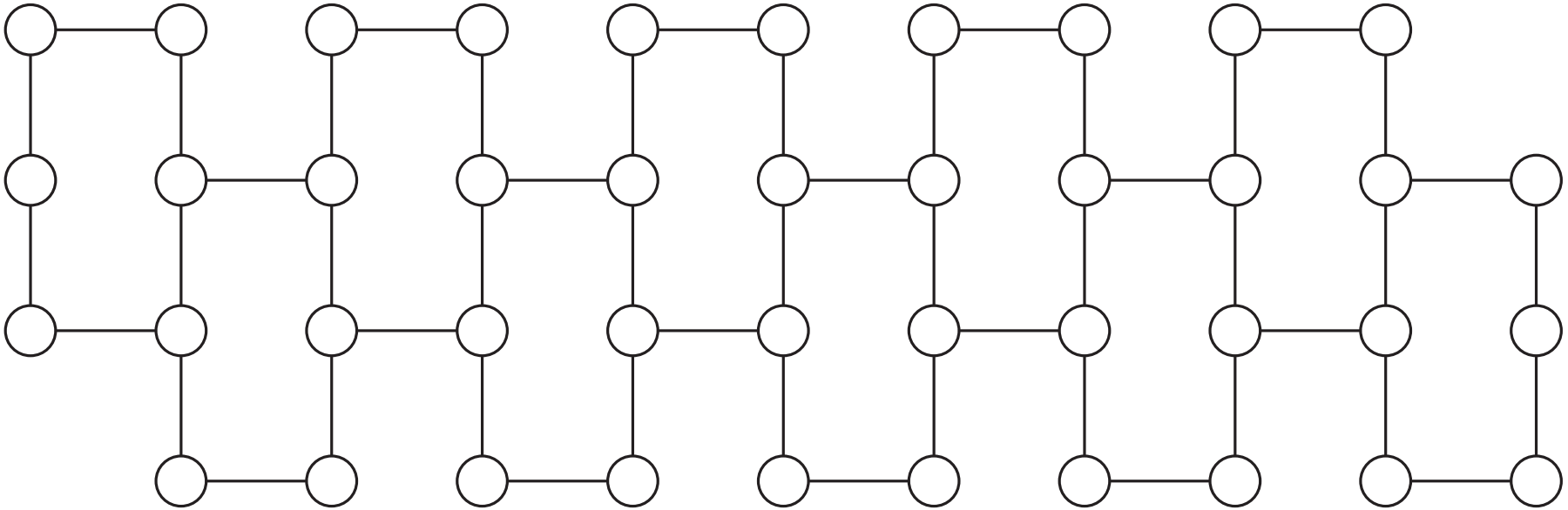
Zigzagring



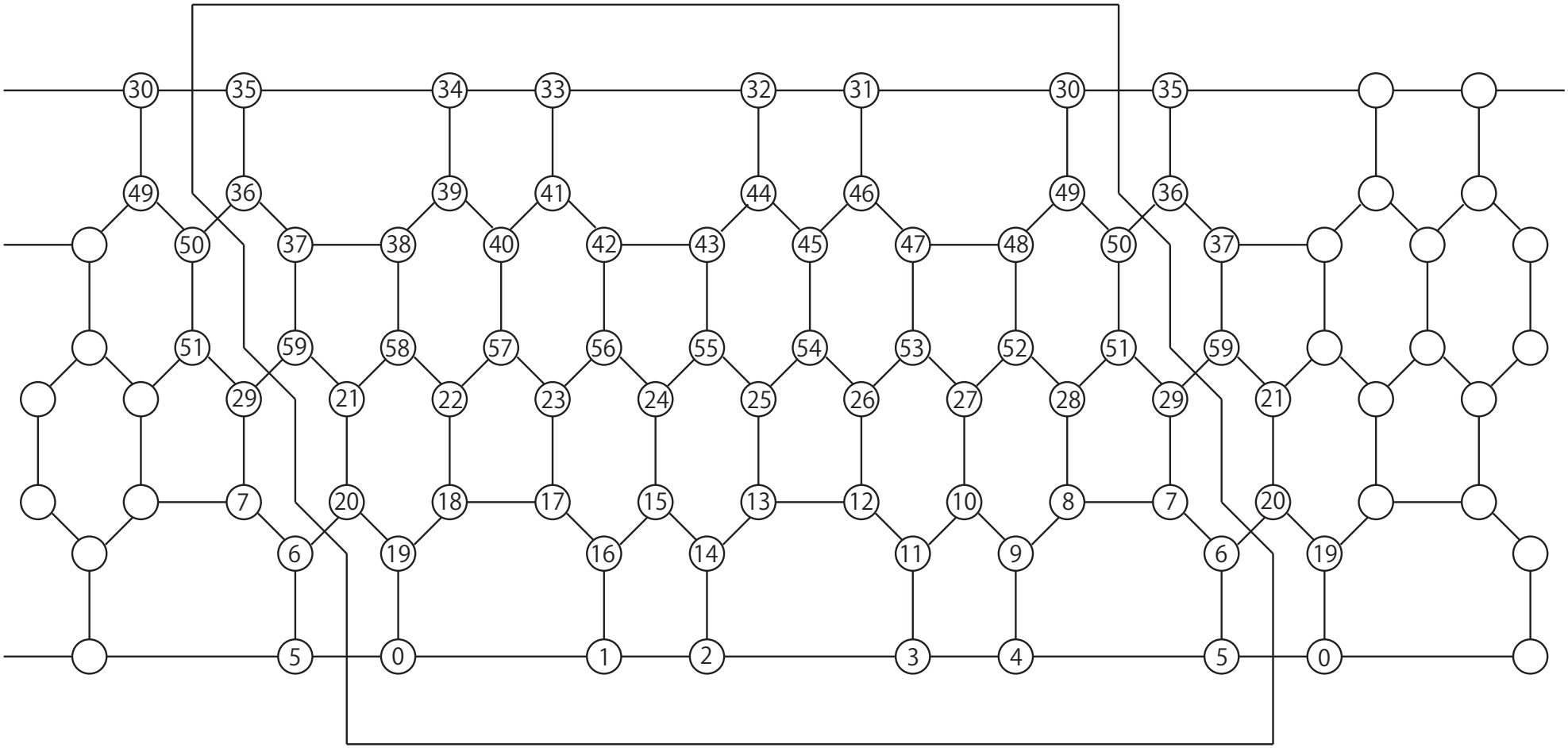
Zigzagline



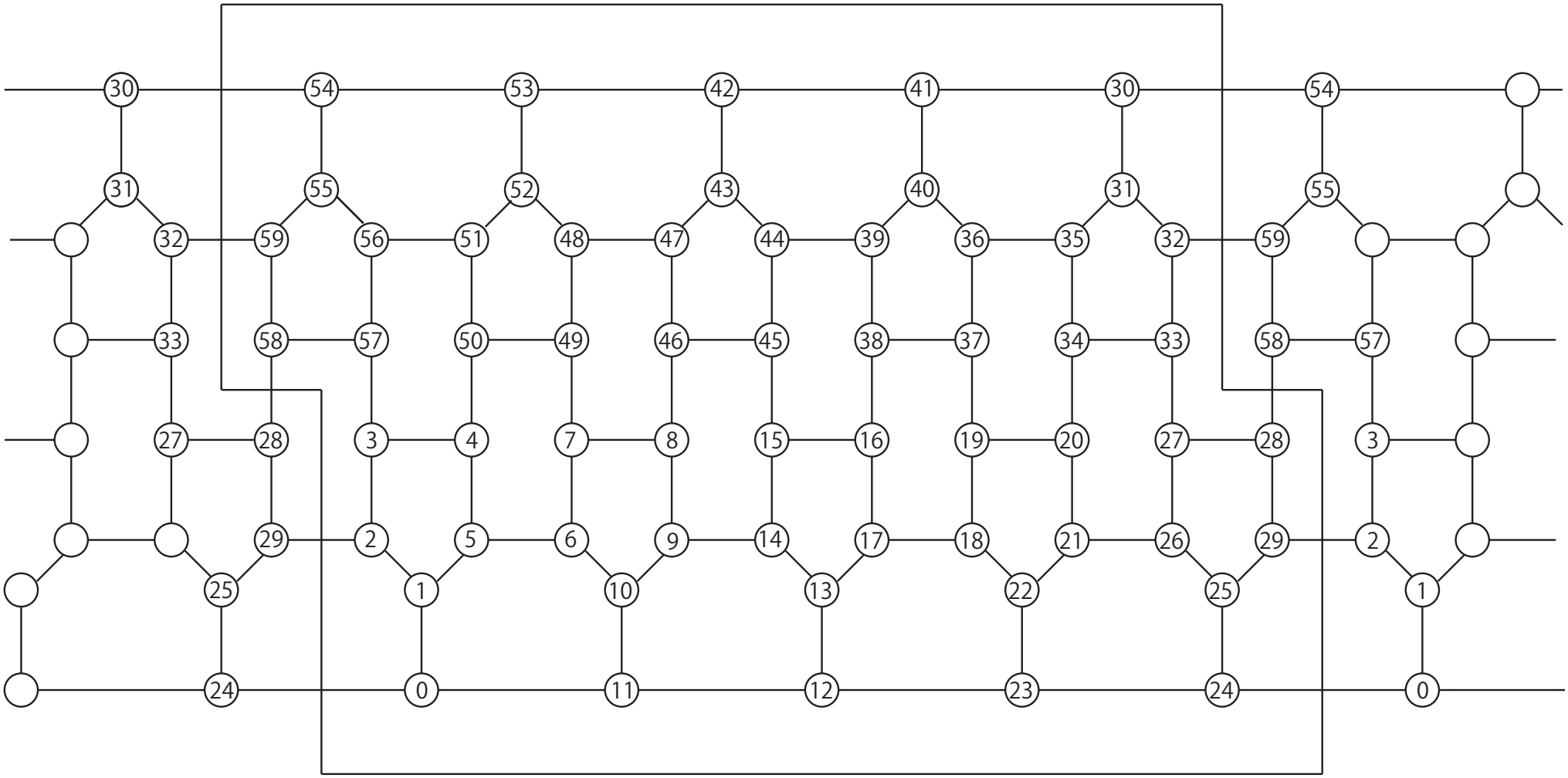
Armchair



A1 (Buckyball, Zigzag)



A1 (Buckyball, Armchair)



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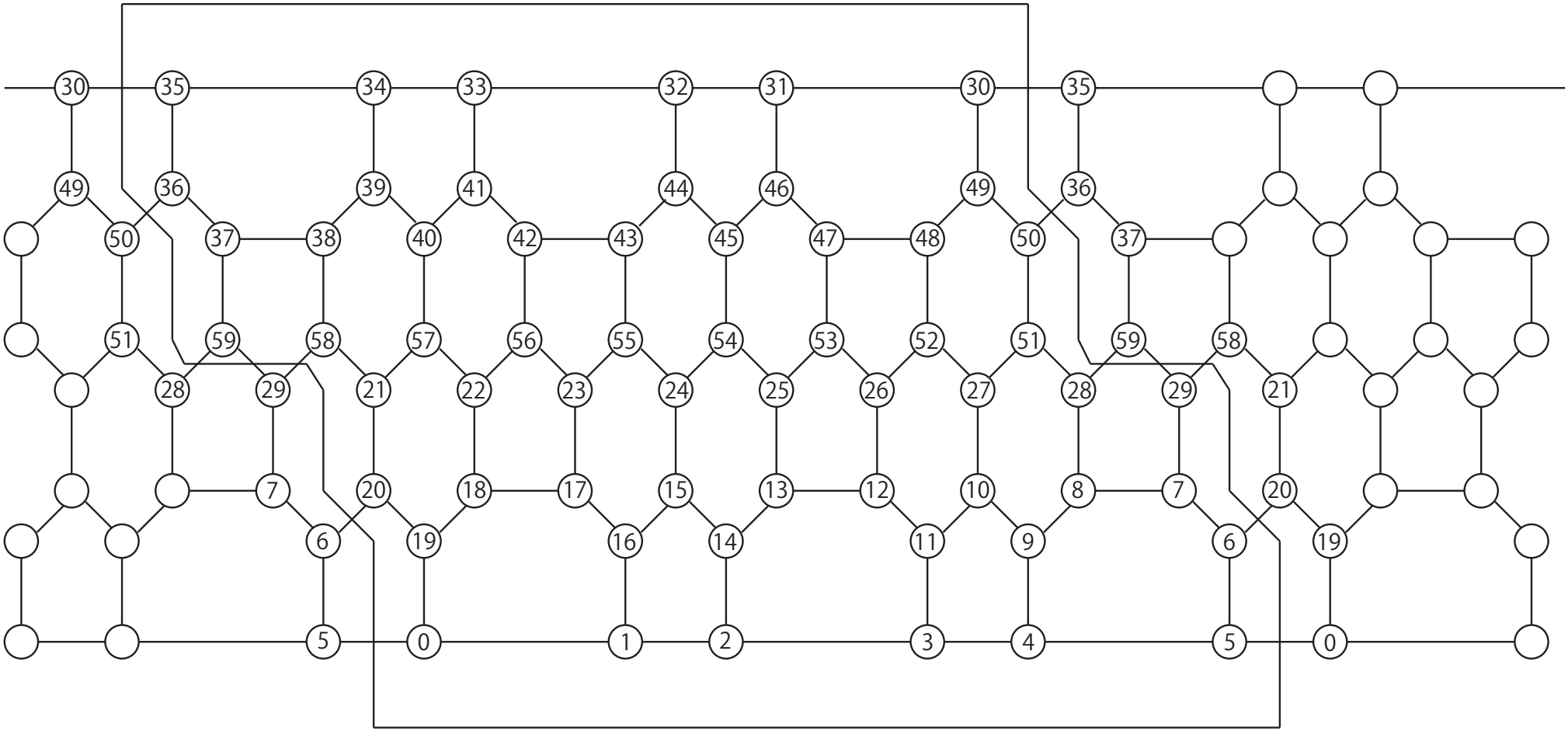
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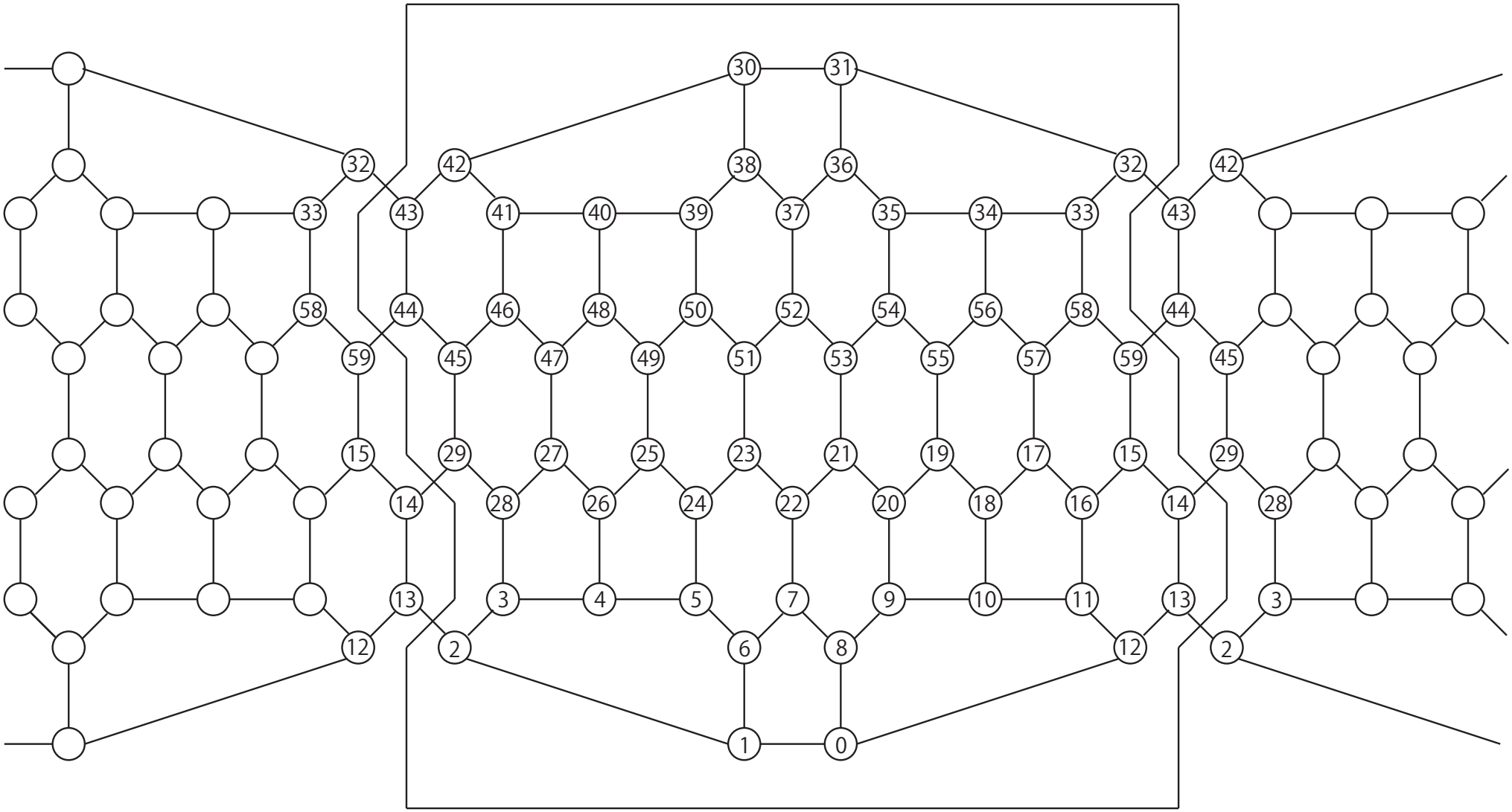
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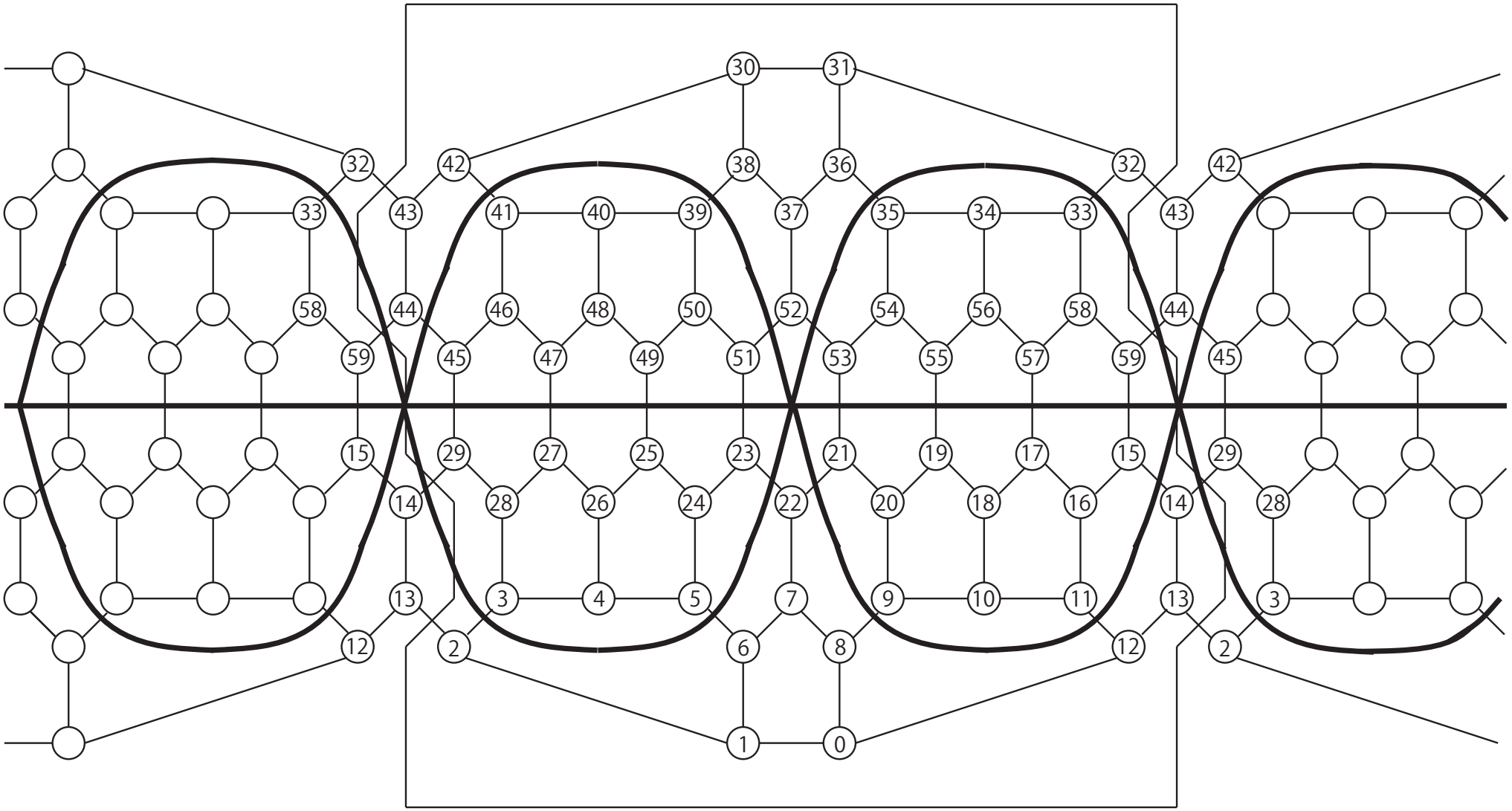
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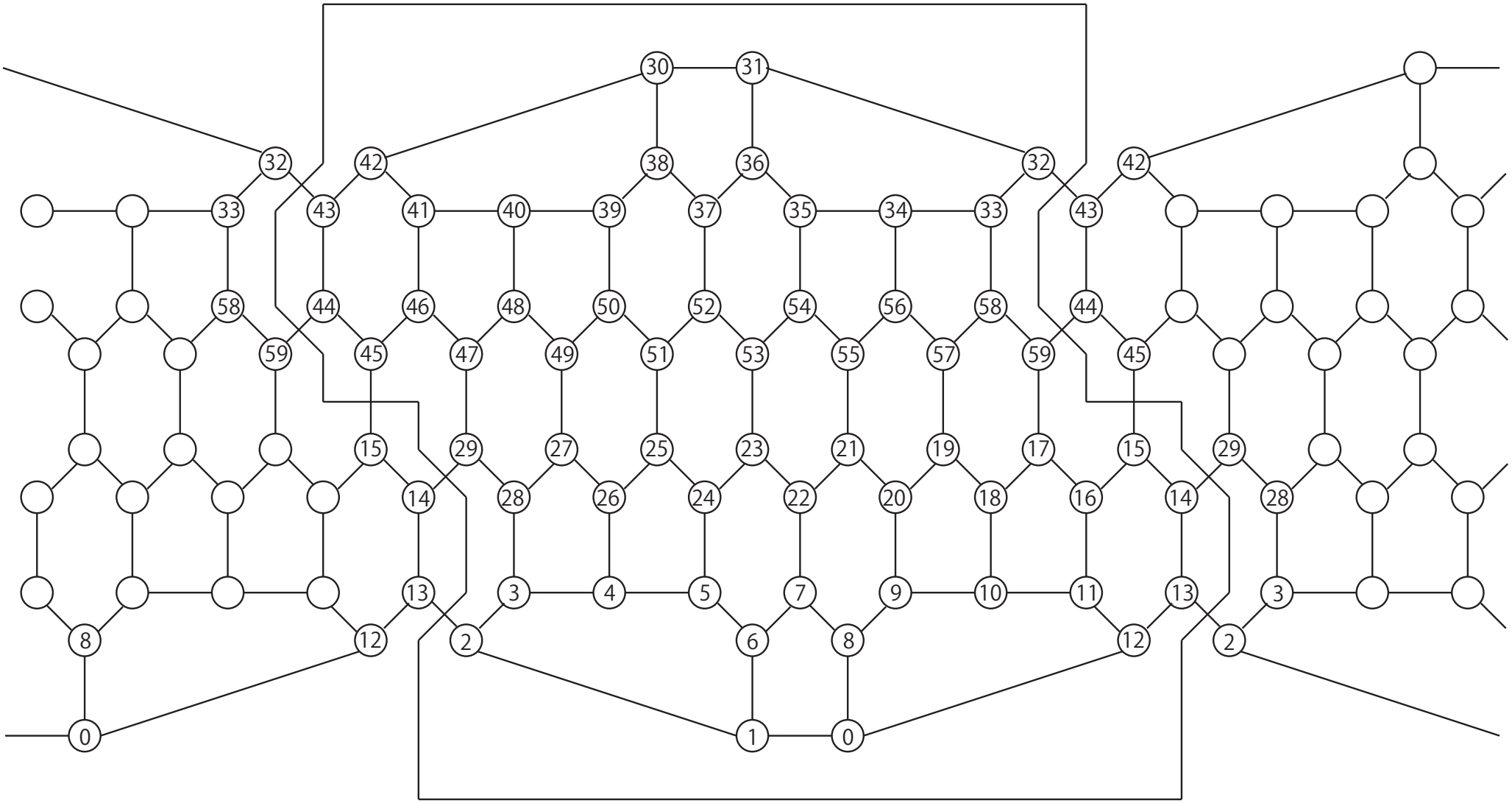
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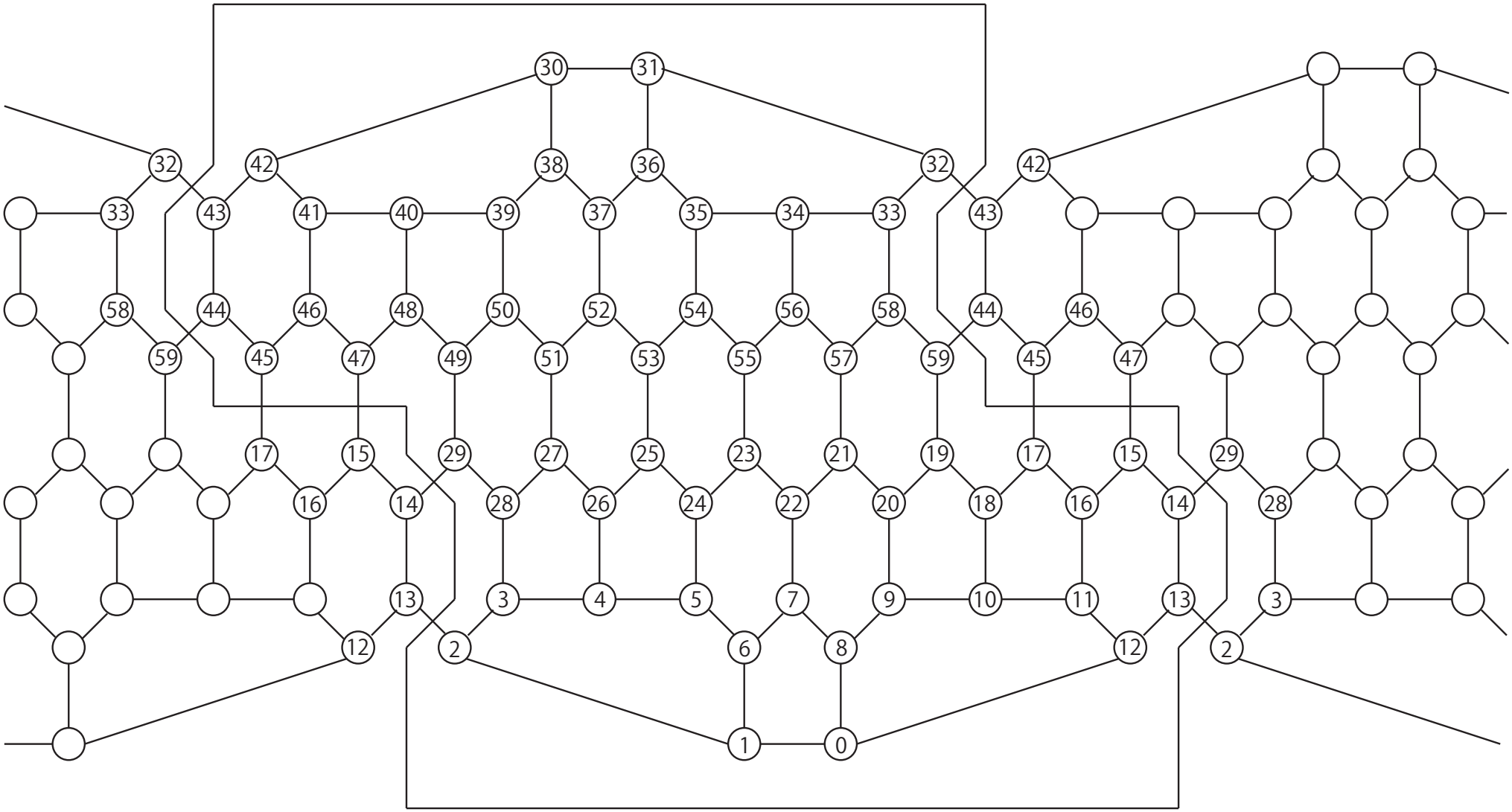
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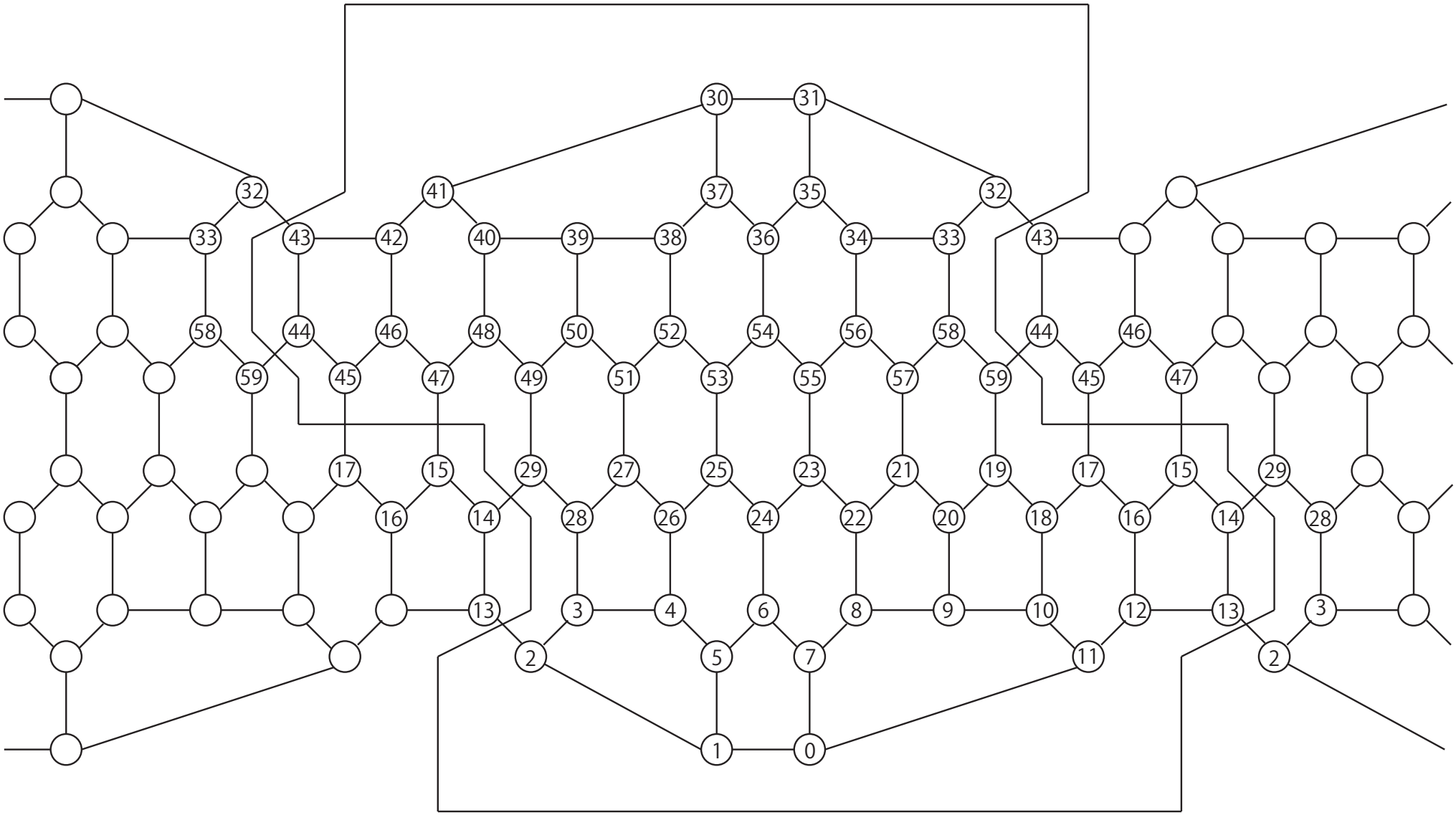
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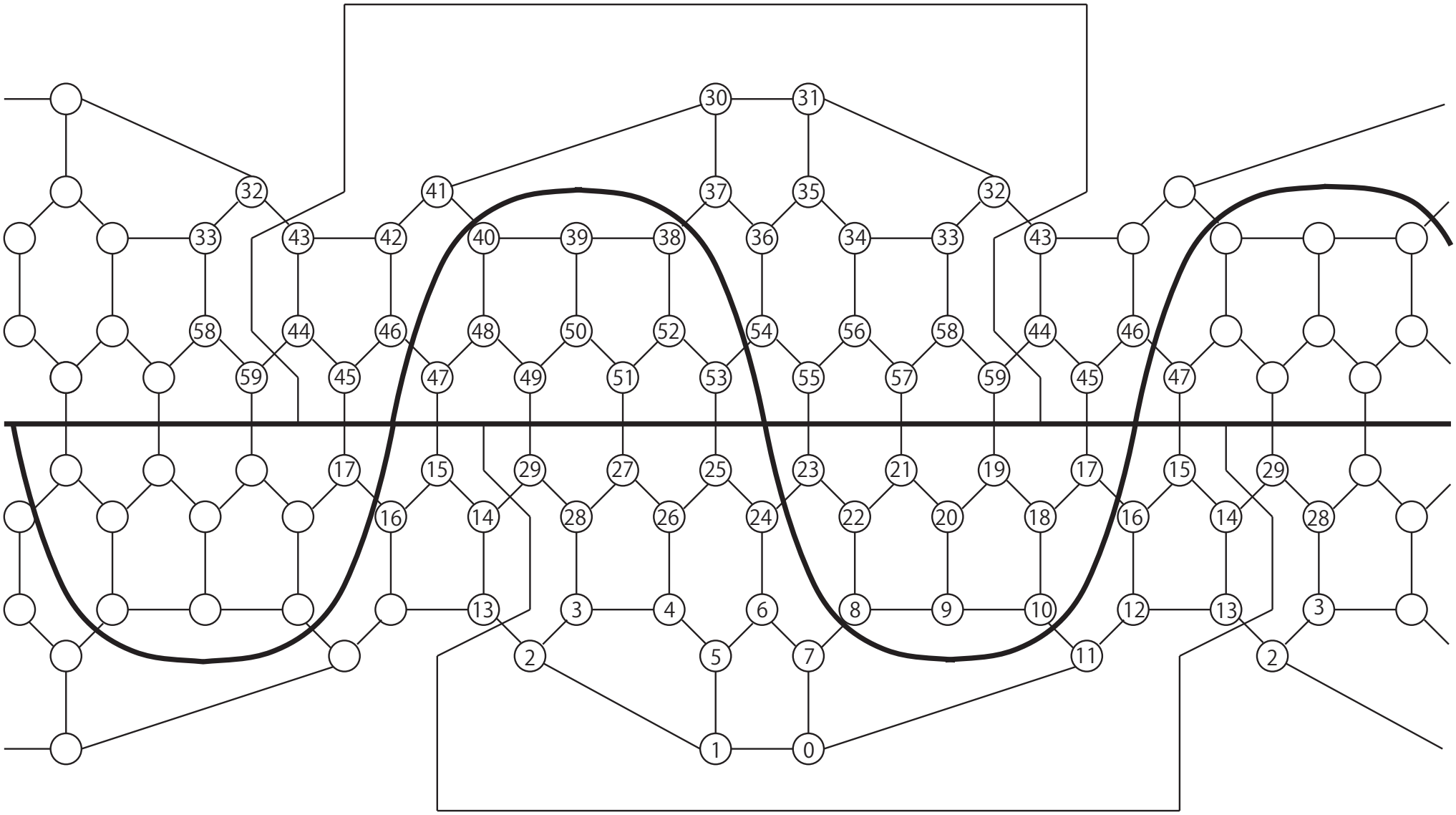
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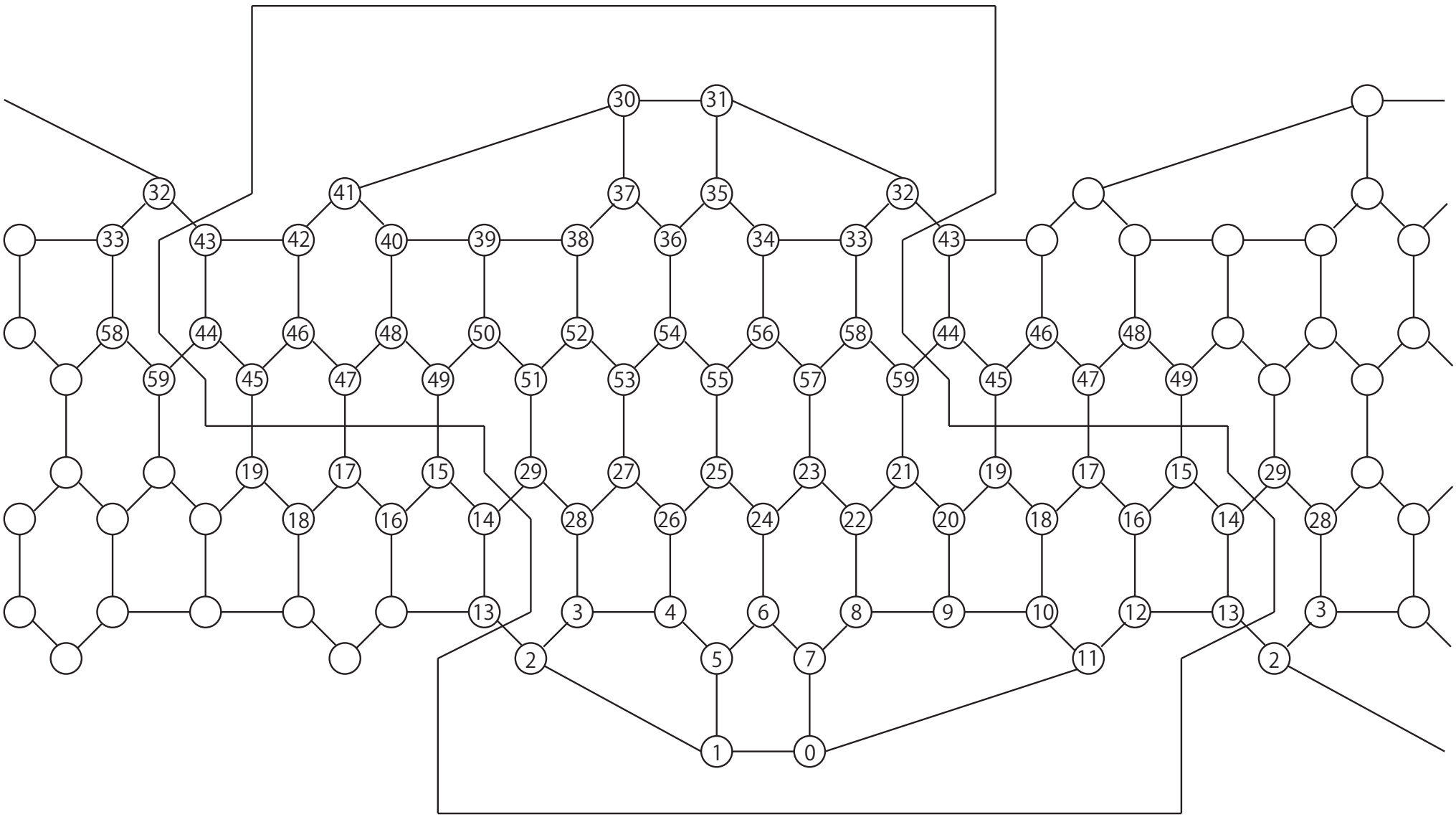
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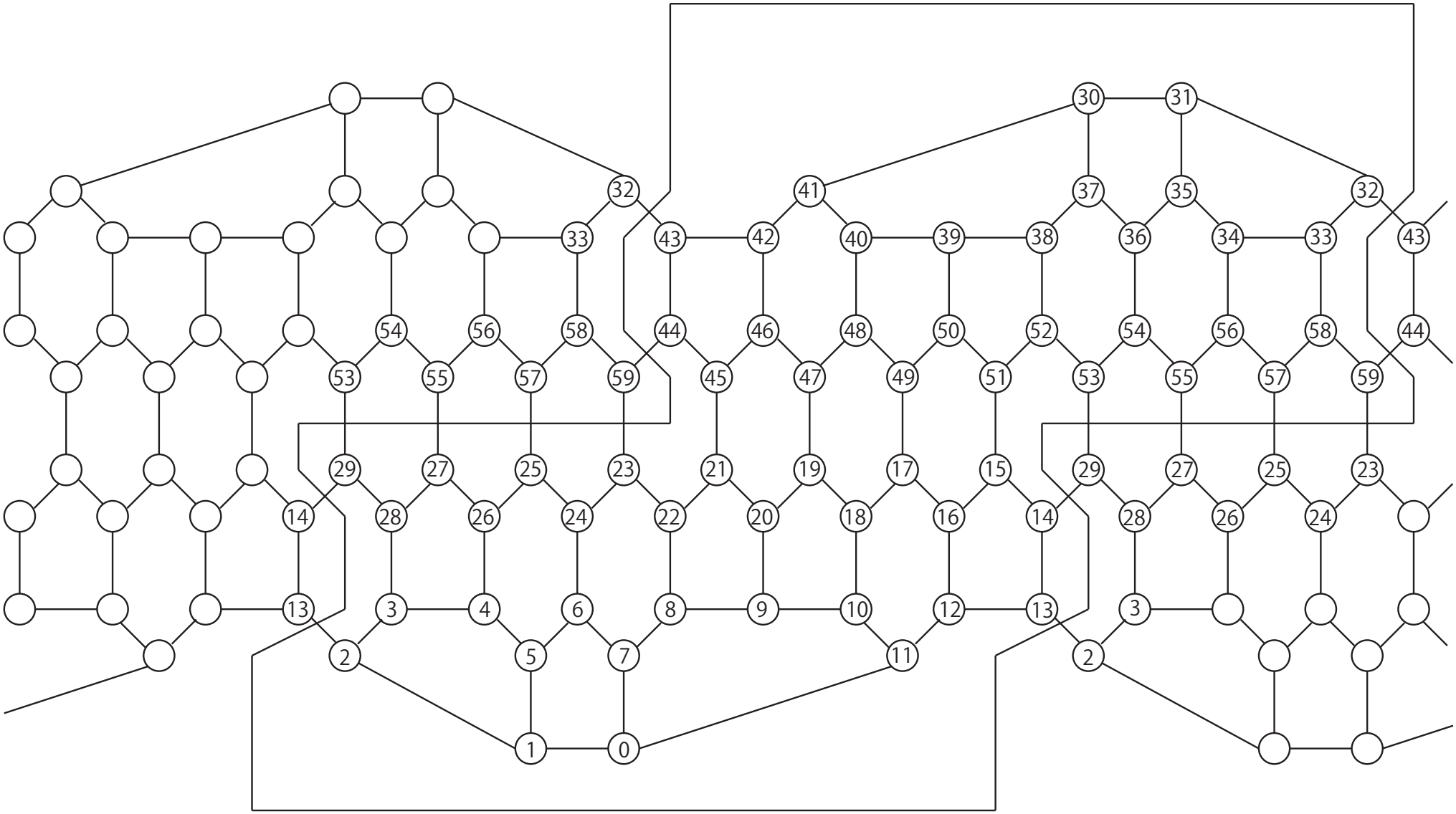
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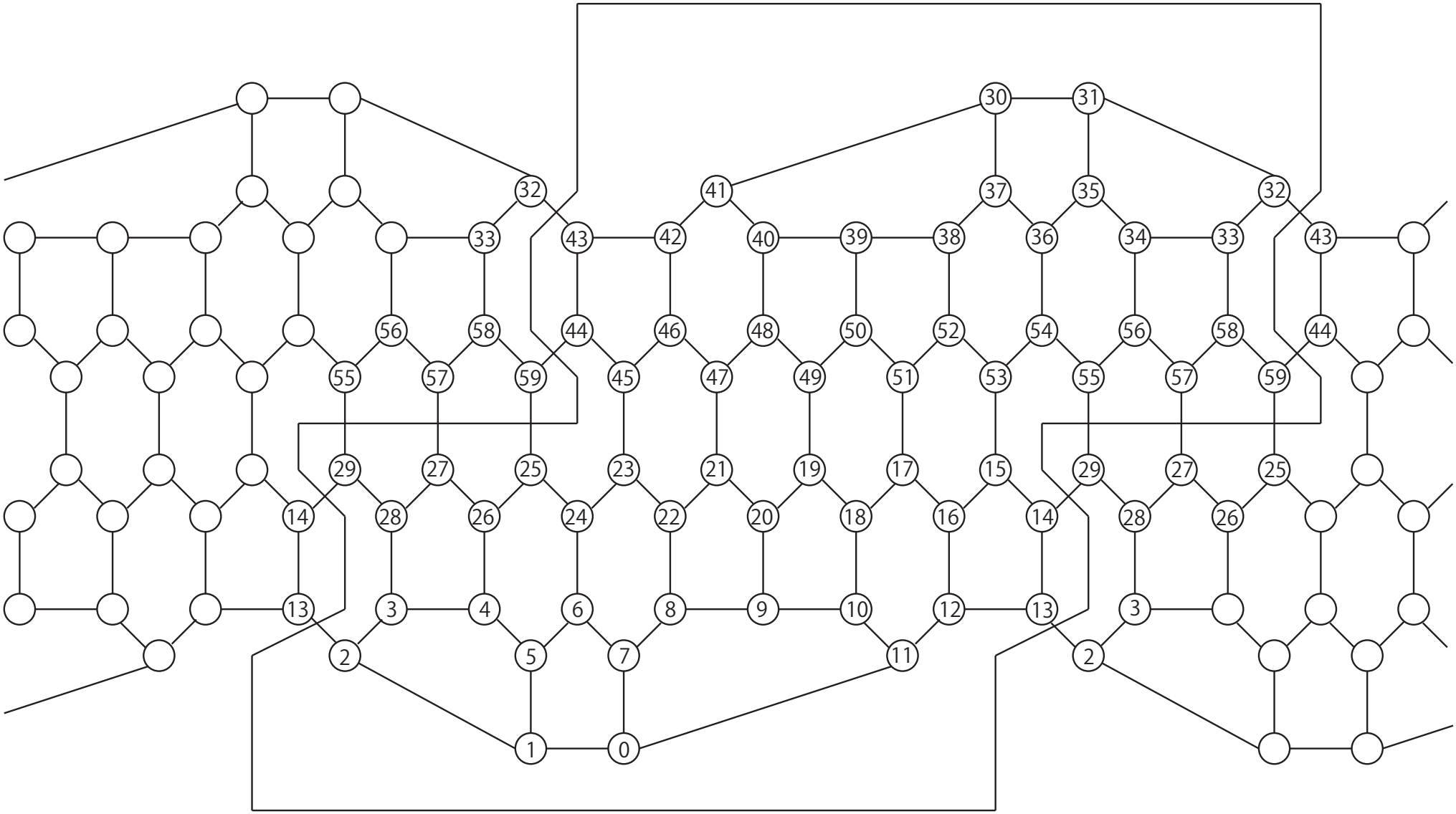
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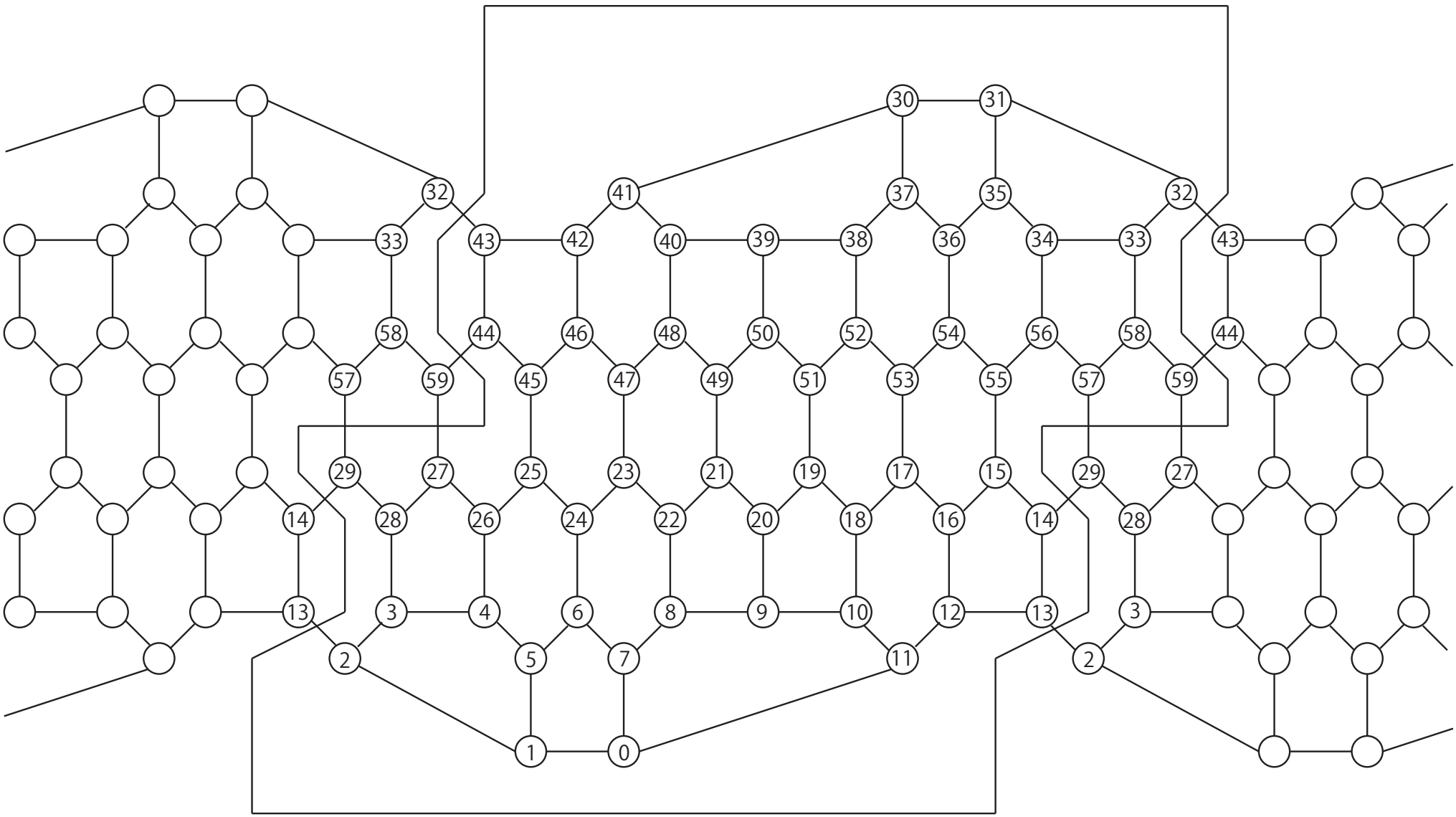
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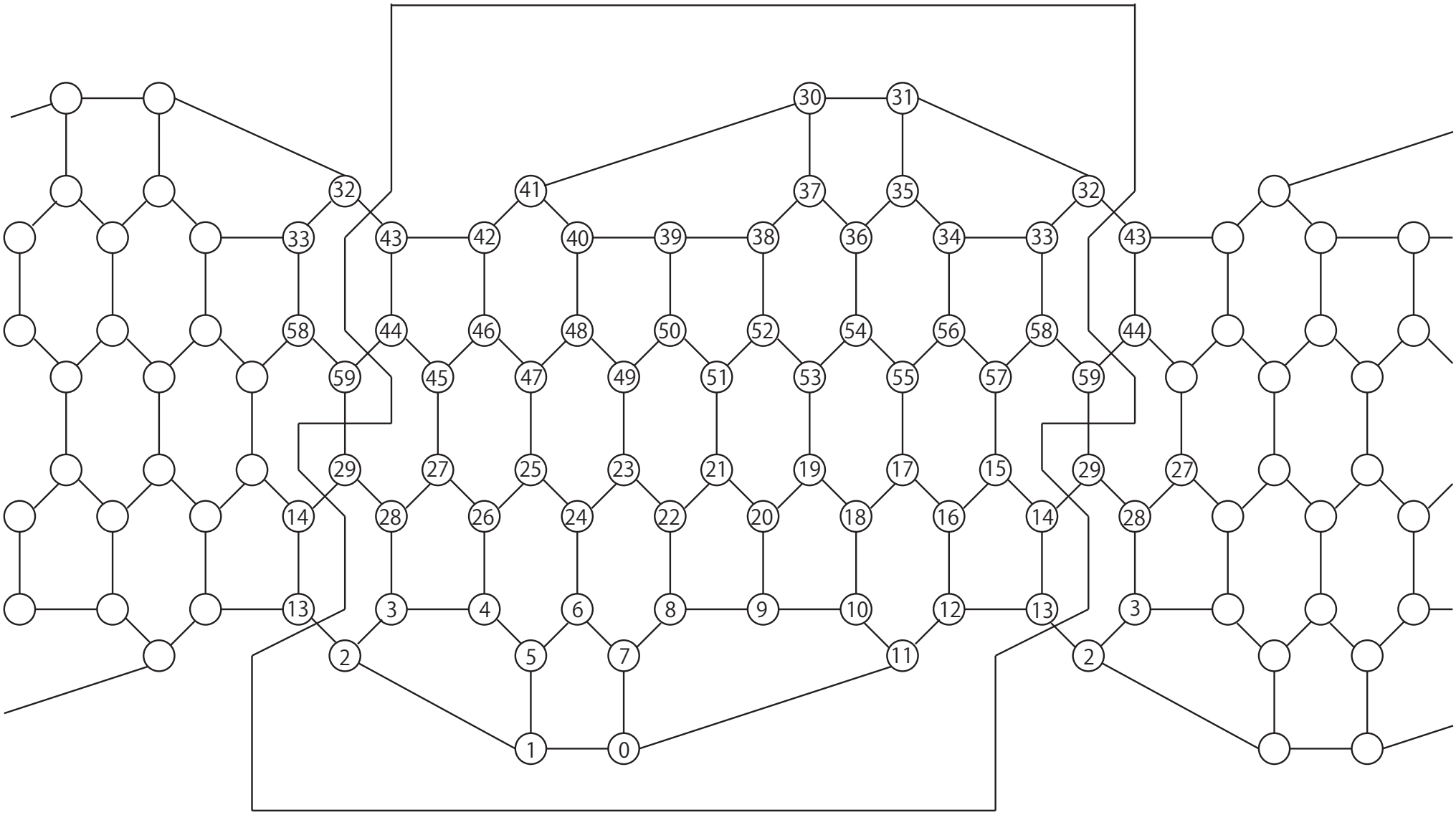
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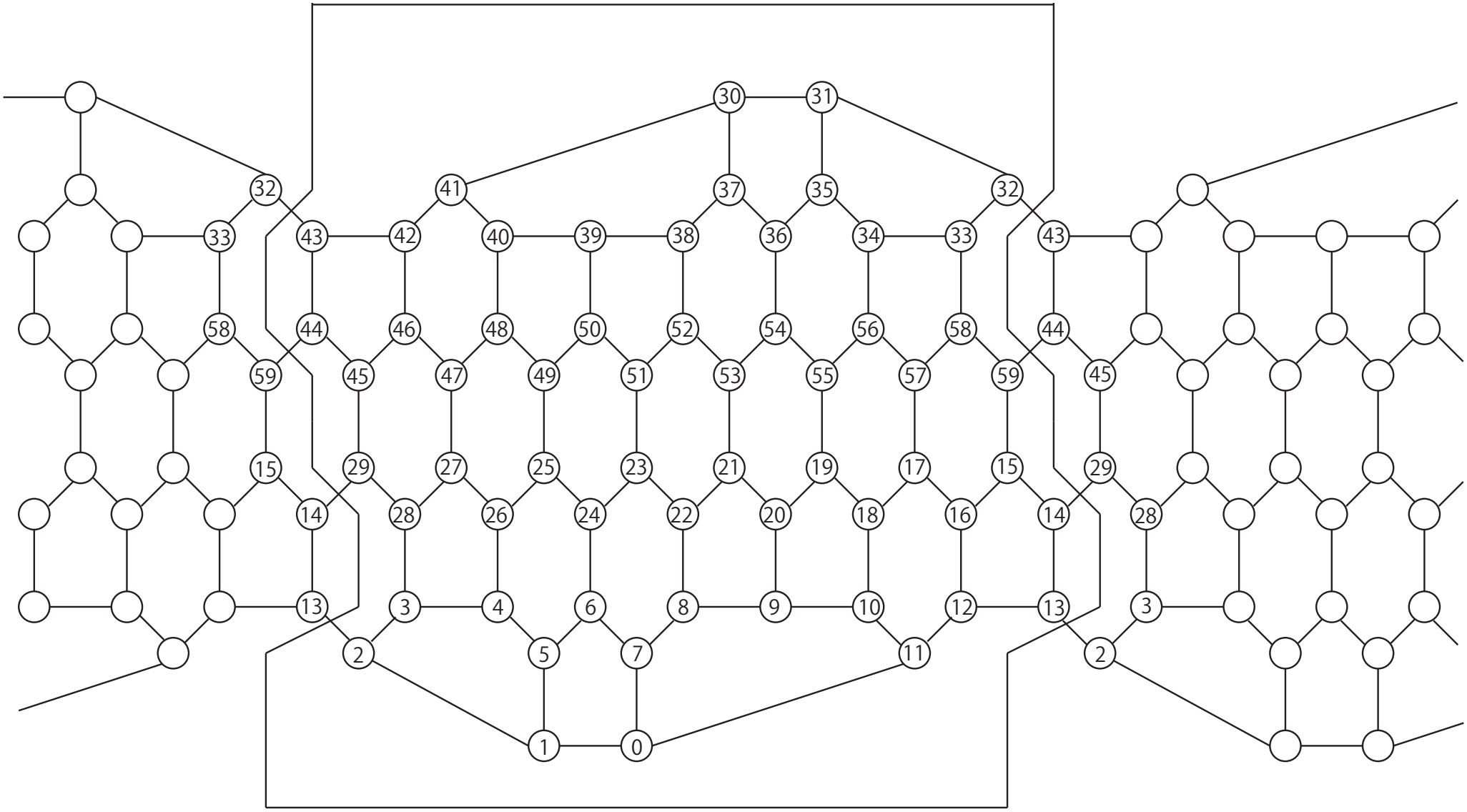
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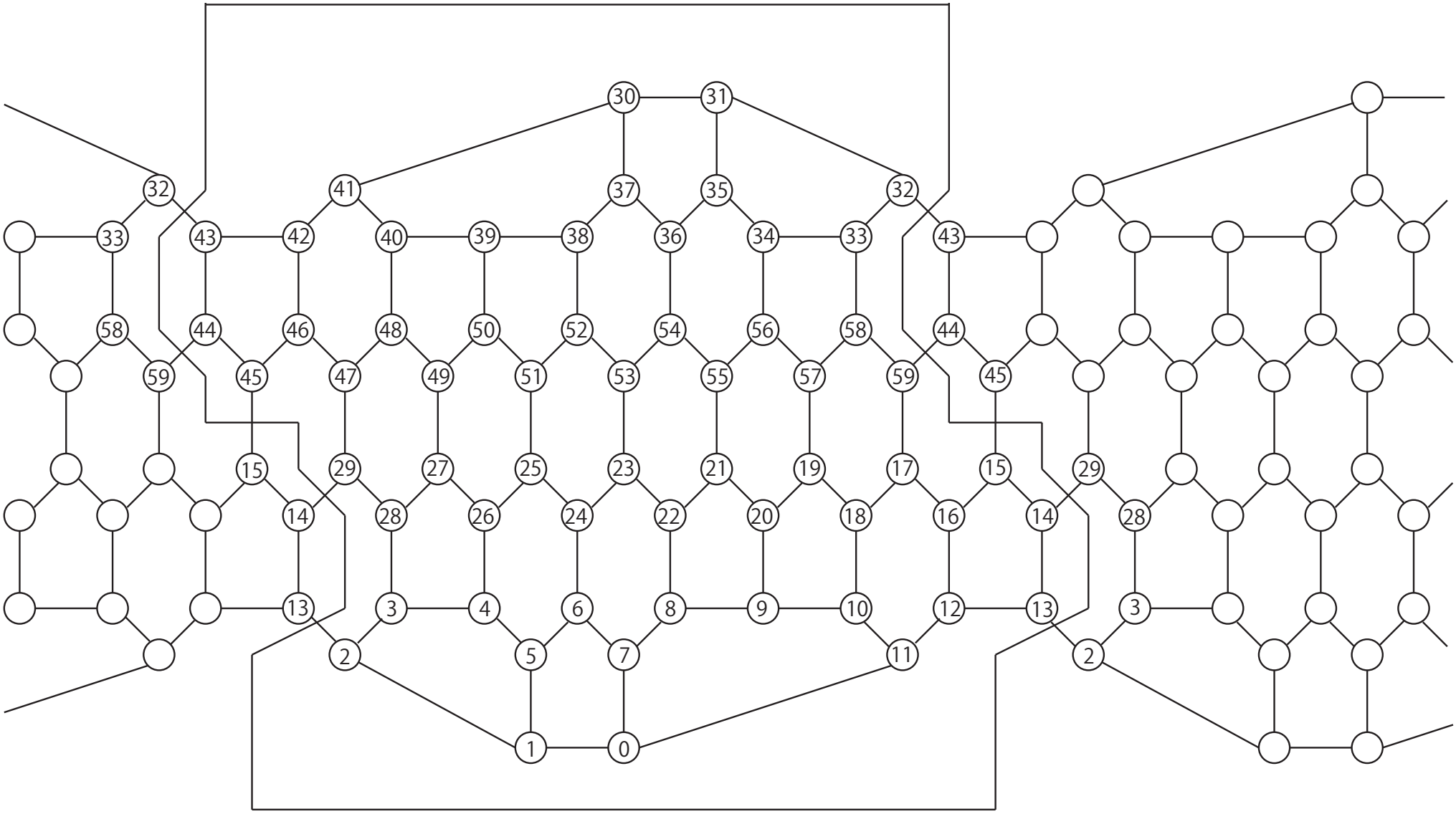
C8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Discrete Laplacian

$$A = \left(\quad a(i,j) \quad \right) \qquad (0 \leq i, j \leq 59)$$

$$a(i,j) = \begin{cases} 3 & (i = j) \\ -1 & (i,j), (j,i) \in e \\ 0 & (\mathsf{else}) \end{cases}$$

A1 Characteristic polynomial

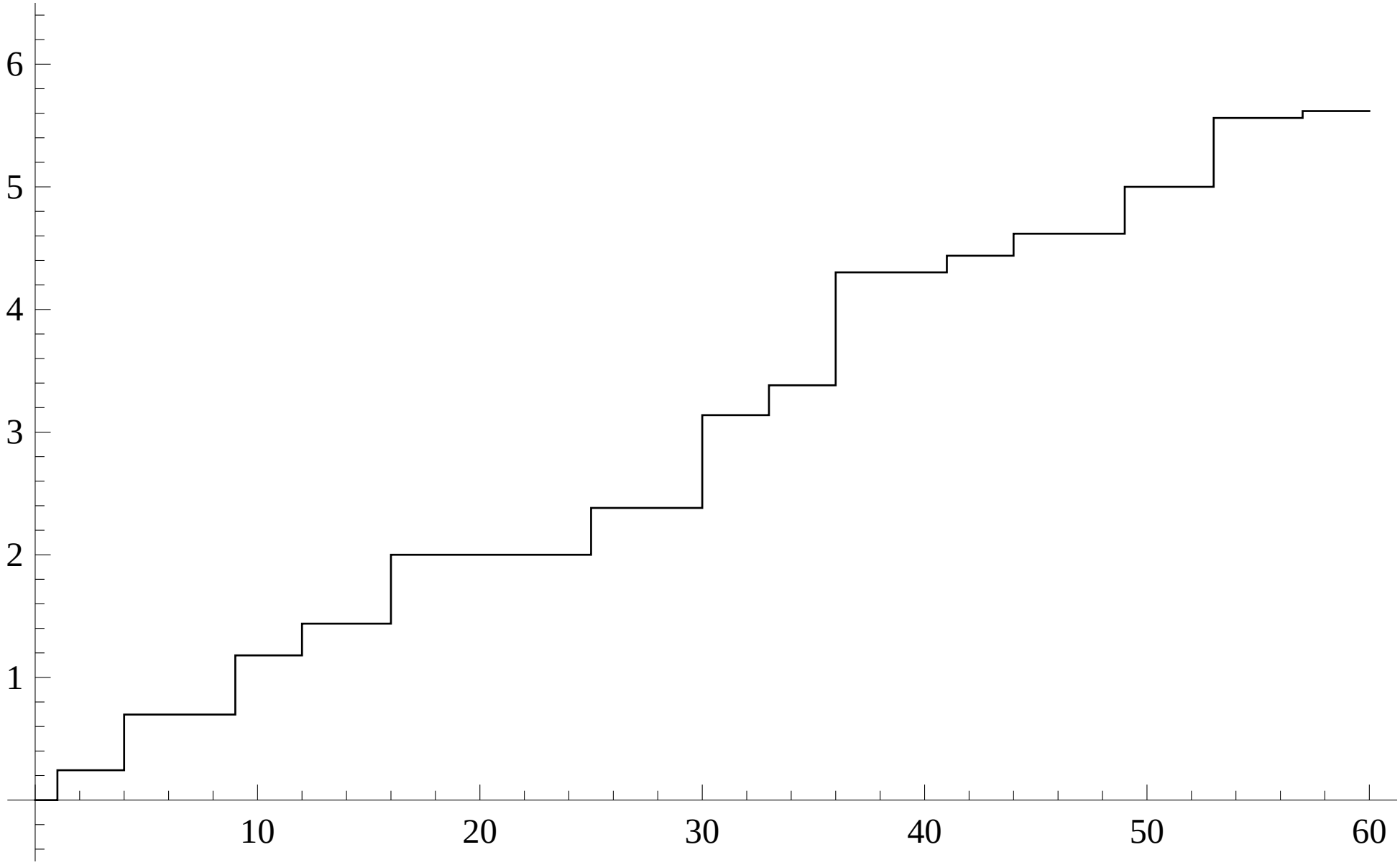
$$P(x) = (-5+x)^4(-2+x)^9 x \left(19-9x+x^2\right)^3 \left(8-7x+x^2\right)^4 \left(11-7x+x^2\right)^5 \left(3-5x+x^2\right)^5 \left(4-22x+25x^2-9x^3+x^4\right)^3$$

A1

$\mathbf{0}$	$=$	$\mathbf{0}$	$\mathbf{(1)}$
$\mathbf{(9-\sqrt{5}-\sqrt{38-2\sqrt{5}})/4}$	$\doteq$	$\mathbf{0.243402}$	$\mathbf{(3)}$
$\mathbf{(5-\sqrt{13})/2}$	$\doteq$	$\mathbf{0.697224}$	$\mathbf{(5)}$
$\mathbf{(9+\sqrt{5}-\sqrt{38+2\sqrt{5}})/4}$	$\doteq$	$\mathbf{1.17975}$	$\mathbf{(3)}$
$\mathbf{(7-\sqrt{17})/2}$	$\doteq$	$\mathbf{1.43845}$	$\mathbf{(4)}$
$\mathbf{2}$	$=$	$\mathbf{2}$	$\mathbf{(9)}$
$\mathbf{(7-\sqrt{5})/2}$	$\doteq$	$\mathbf{2.38197}$	$\mathbf{(5)}$
$\mathbf{(9-\sqrt{5}+\sqrt{38-2\sqrt{5}})/4}$	$\doteq$	$\mathbf{3.13856}$	$\mathbf{(3)}$
$\mathbf{(9-\sqrt{5})/2}$	$\doteq$	$\mathbf{3.38197}$	$\mathbf{(3)}$
$\mathbf{(5+\sqrt{13})/2}$	$\doteq$	$\mathbf{4.30278}$	$\mathbf{(5)}$
$\mathbf{(9+\sqrt{5}+\sqrt{38+2\sqrt{5}})/4}$	$\doteq$	$\mathbf{4.43828}$	$\mathbf{(3)}$
$\mathbf{(7+\sqrt{5})/2}$	$\doteq$	$\mathbf{4.61803}$	$\mathbf{(5)}$
$\mathbf{5}$	$=$	$\mathbf{5}$	$\mathbf{(4)}$
$\mathbf{(7+\sqrt{17})/2}$	$\doteq$	$\mathbf{5.56155}$	$\mathbf{(4)}$
$\mathbf{(9+\sqrt{5})/2}$	$\doteq$	$\mathbf{5.61803}$	$\mathbf{(3)}$

A1	Eigenvalue (Multiplicity)
0	(1)
0.243402	(3)
0.697224	(5)
1.17975	(3)
1.43845	(4)
2	(9)
2.38197	(5)
3.13856	(3)
3.38197	(3)
4.30278	(5)
4.43828	(3)
4.61803	(5)
5	(4)
5.56155	(4)
5.61803	(3)

A1 Eigenvalue distribution



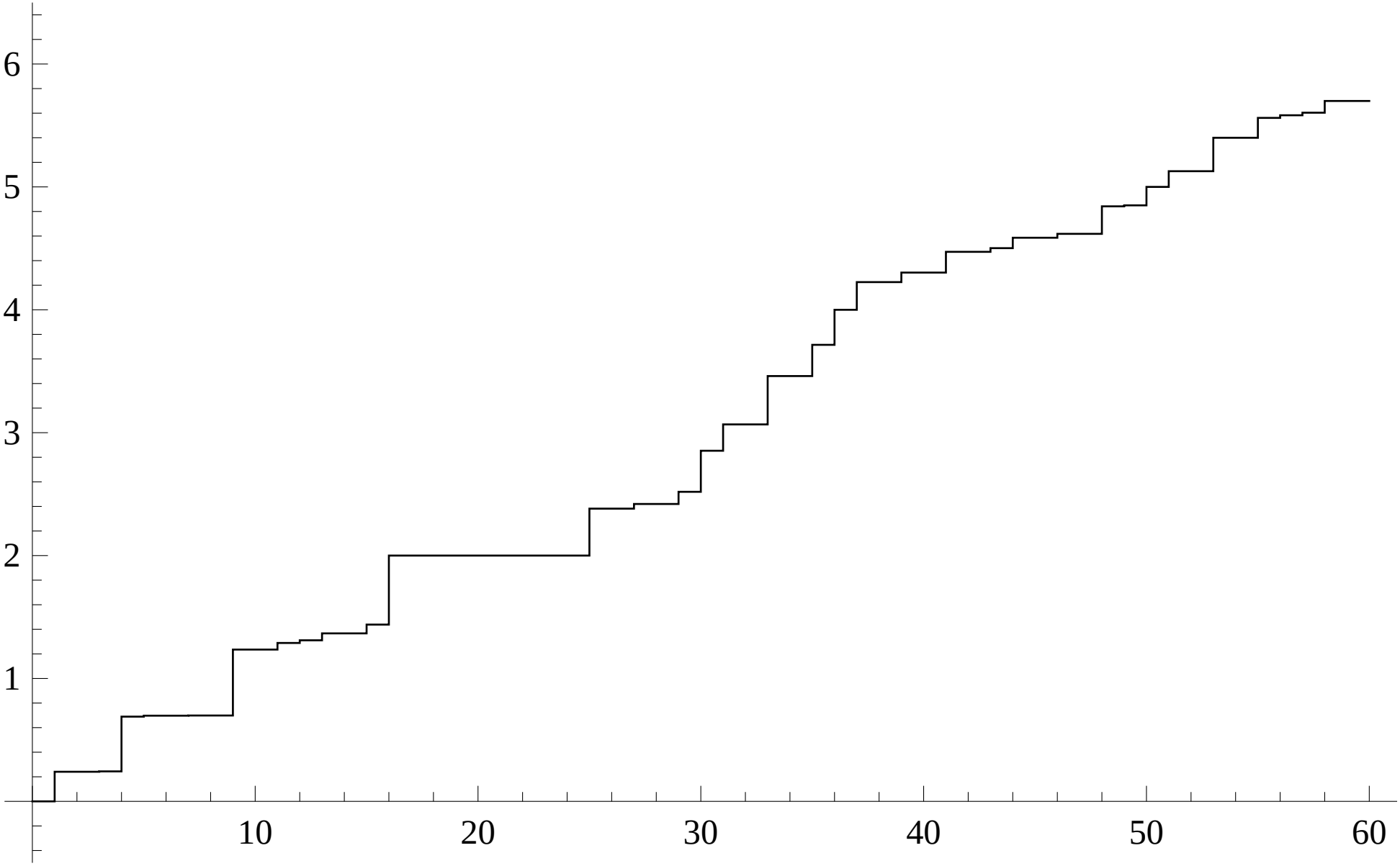
A2

$$P(x) = (-5+x)(-4+x)(-2+x)^9 x$$
$$\left(8-7x+x^2\right)\left(11-7x+x^2\right)^2\left(3-5x+x^2\right)^2$$
$$\left(98-575x+821x^2-487x^3+139x^4-19x^5+x^6\right)$$
$$\left(256-827x+955x^2-515x^3+141x^4-19x^5+x^6\right)$$
$$(-99848+937037x-3344594x^2+6391936x^3-$$
$$7532510x^4+5894944x^5-3195866x^6+1227895x^7-$$
$$337050x^8+65701x^9-8886x^{10}+793x^{11}-42x^{12}+x^{13})^2$$

A2

0	(1)	2.38197	(2)	4.58638	(2)
0.240977	(2)	2.41983	(2)	4.61803	(2)
0.243925	(1)	2.51894	(1)	4.84189	(1)
0.689504	(1)	2.85308	(1)	4.84987	(1)
0.697224	(2)	3.06739	(2)	5	(1)
0.698544	(2)	3.46082	(2)	5.12782	(2)
1.23524	(2)	3.71489	(1)	5.3998	(2)
1.28896	(1)	4	(1)	5.56155	(1)
1.31072	(1)	4.2253	(2)	5.58296	(1)
1.36696	(2)	4.30278	(2)	5.6037	(1)
1.43845	(1)	4.47167	(2)	5.69926	(2)
2	(9)	4.50157	(1)		

A2



B1

$$P(x) = (-4+x)^3(-3+x)^2(-2+x)^3x$$

$$\left(14-8x+x^2\right)\left(6-6x+x^2\right)\left(-46+44x-12x^2+x^3\right)$$

$$\left(-17+23x-9x^2+x^3\right)^2\left(-13+23x-9x^2+x^3\right)^2$$

$$\left(-10+18x-8x^2+x^3\right)\left(-2+8x-6x^2+x^3\right)$$

$$\left(64-118x+66x^2-14x^3+x^4\right)$$

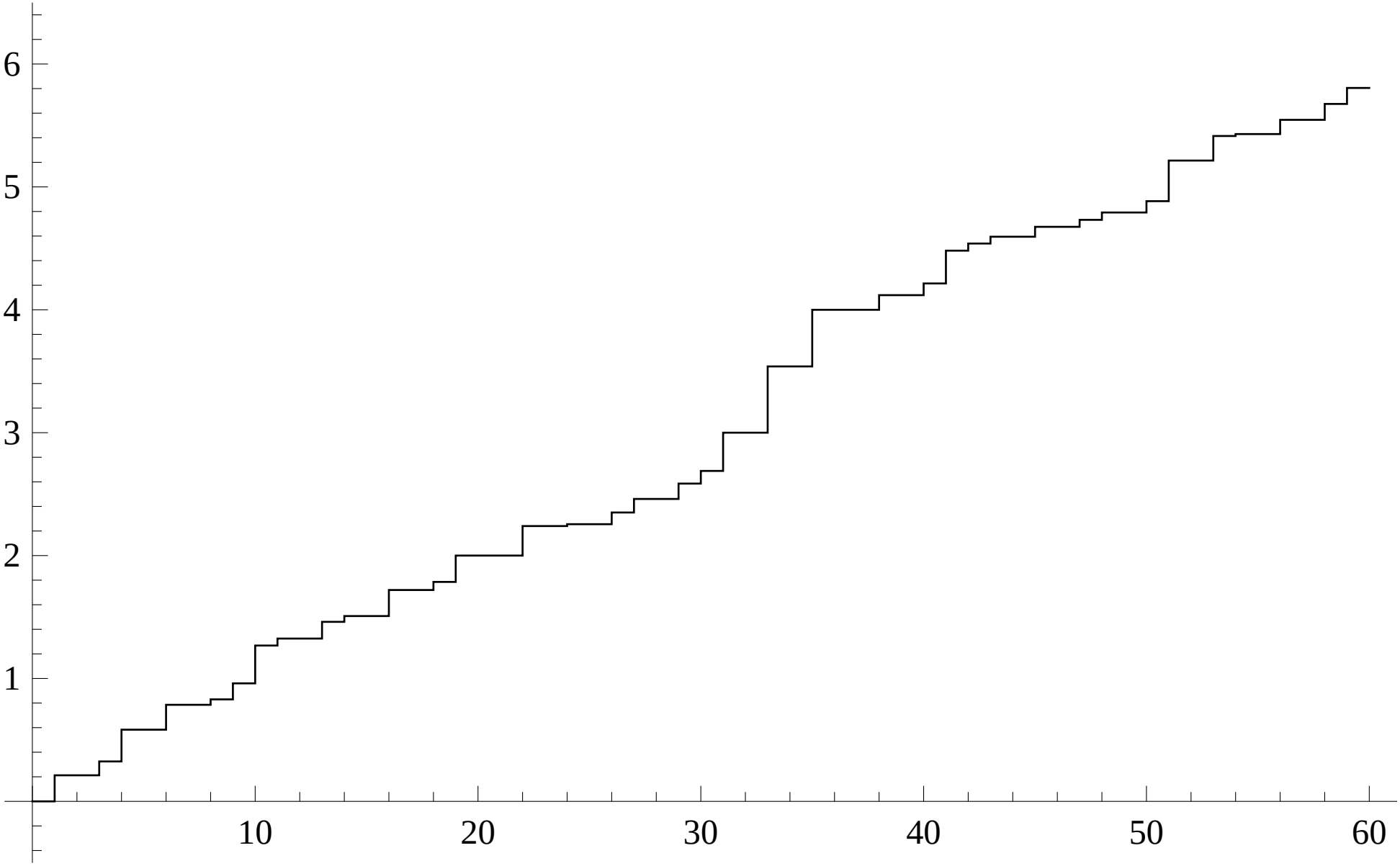
$$\left(-18+112x-142x^2+69x^3-14x^4+x^5\right)^2$$

$$\left(246-830x+956x^2-515x^3+141x^4-19x^5+x^6\right)^2$$

B1

0	(1)	2	(3)	4.53919	(1)
0.21214	(2)	2.24021	(2)	4.59452	(2)
0.324869	(1)	2.25546	(2)	4.67513	(2)
0.583305	(2)	2.3506	(1)	4.73205	(1)
0.78568	(2)	2.46081	(2)	4.79183	(2)
0.829914	(1)	2.58579	(1)	4.88395	(1)
0.960327	(1)	2.68889	(1)	5.21432	(2)
1.26795	(1)	3	(2)	5.41421	(1)
1.32487	(2)	3.53919	(2)	5.42996	(2)
1.46081	(1)	4	(3)	5.54563	(2)
1.50792	(2)	4.1192	(2)	5.67513	(1)
1.71983	(2)	4.21432	(1)	5.80513	(1)
1.78568	(1)	4.48119	(1)		

B1



B2

$$P(x) = (-4+x)(-2+x)^2 x$$

$$\left(-44768+455938x-1734518x^2+3527864x^3-4429788x^4+\right.$$
$$3698064x^5-2139272x^6+876477x^7-256170x^8+53055x^9-$$
$$\left.7604x^{10}+717x^{11}-40x^{12}+x^{13}\right)$$

$$\left(-167880+1135190x-3333882x^2+5657352x^3-6220108x^4+\right.$$
$$4701136x^5-2523260x^6+977337x^7-274050x^8+55099x^9-$$
$$\left.7740x^{10}+721x^{11}-40x^{12}+x^{13}\right)$$

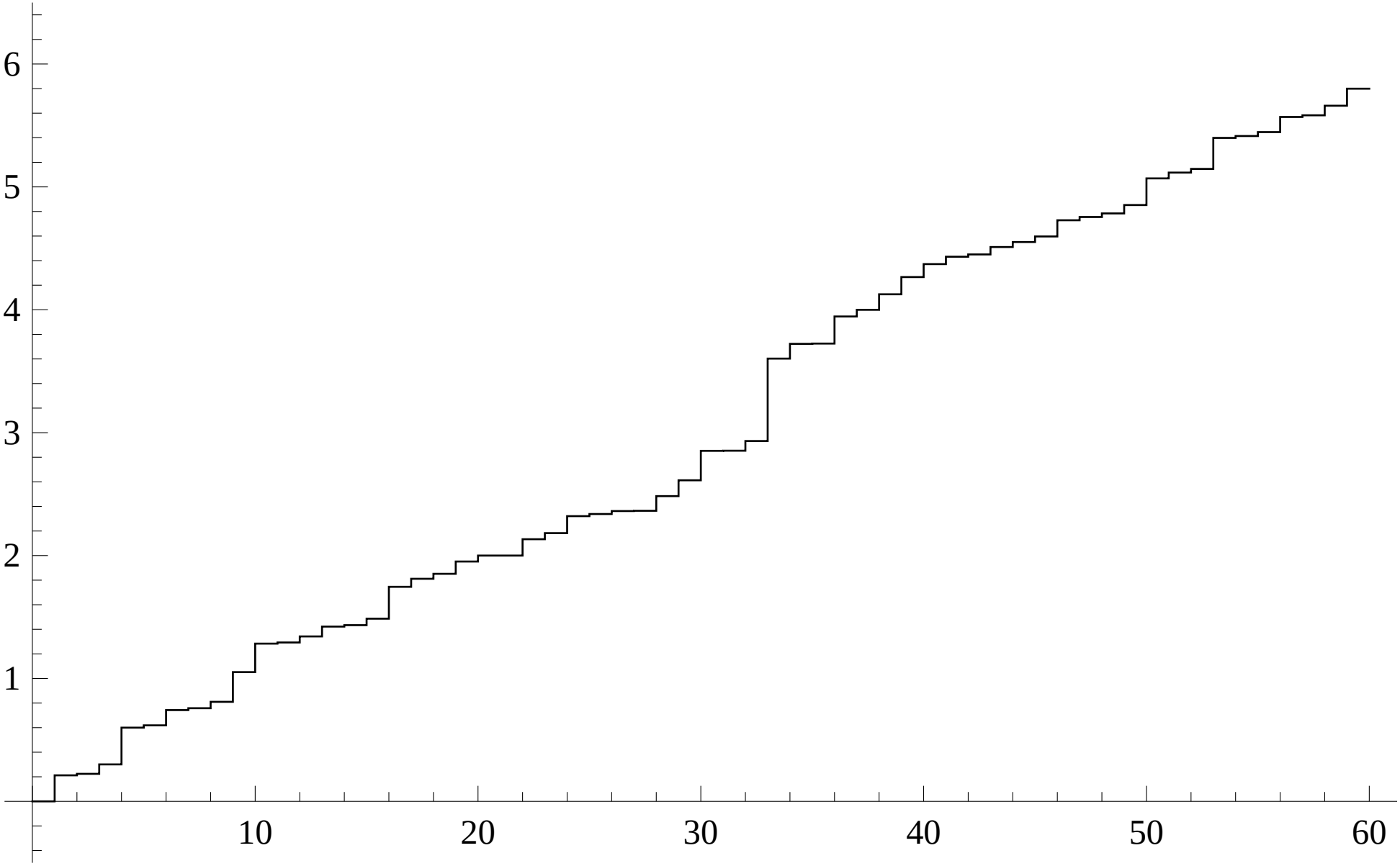
$$\left(-398720+4215458x-17515520x^2+40405876x^3-59577850x^4+\right.$$
$$60447322x^5-44042916x^6+23631964x^7-9463582x^8+2839853x^9-$$
$$\left.635464x^{10}+104419x^{11}-12226x^{12}+965x^{13}-46x^{14}+x^{15}\right)$$

$$\left(-457320+4434850x-17873280x^2+40737960x^3-59770398x^4+\right.$$
$$60519398x^5-44060300x^6+23634568x^7-9463802x^8+2839861x^9-$$
$$\left.635464x^{10}+104419x^{11}-12226x^{12}+965x^{13}-46x^{14}+x^{15}\right)$$

B2

0	(1)	1.48656	(1)	2.85334	(1)	4.7286	(1)
0.211901	(1)	1.7458	(1)	2.93216	(1)	4.75496	(1)
0.224085	(1)	1.81129	(1)	3.6029	(1)	4.78418	(1)
0.300792	(1)	1.85173	(1)	3.72296	(1)	4.85239	(1)
0.599828	(1)	1.95162	(1)	3.72526	(1)	5.06915	(1)
0.618574	(1)	2	(2)	3.94554	(1)	5.11649	(1)
0.742765	(1)	2.13315	(1)	4	(1)	5.1464	(1)
0.758006	(1)	2.18264	(1)	4.12625	(1)	5.39871	(1)
0.810056	(1)	2.3209	(1)	4.26628	(1)	5.41398	(1)
1.05183	(1)	2.33825	(1)	4.37178	(1)	5.44603	(1)
1.28356	(1)	2.36237	(1)	4.43193	(1)	5.56901	(1)
1.29283	(1)	2.36446	(1)	4.45048	(1)	5.58262	(1)
1.34215	(1)	2.48381	(1)	4.51069	(1)	5.6606	(1)
1.42229	(1)	2.61264	(1)	4.55139	(1)	5.79924	(1)
1.43387	(1)	2.85227	(1)	4.59665	(1)		

B2



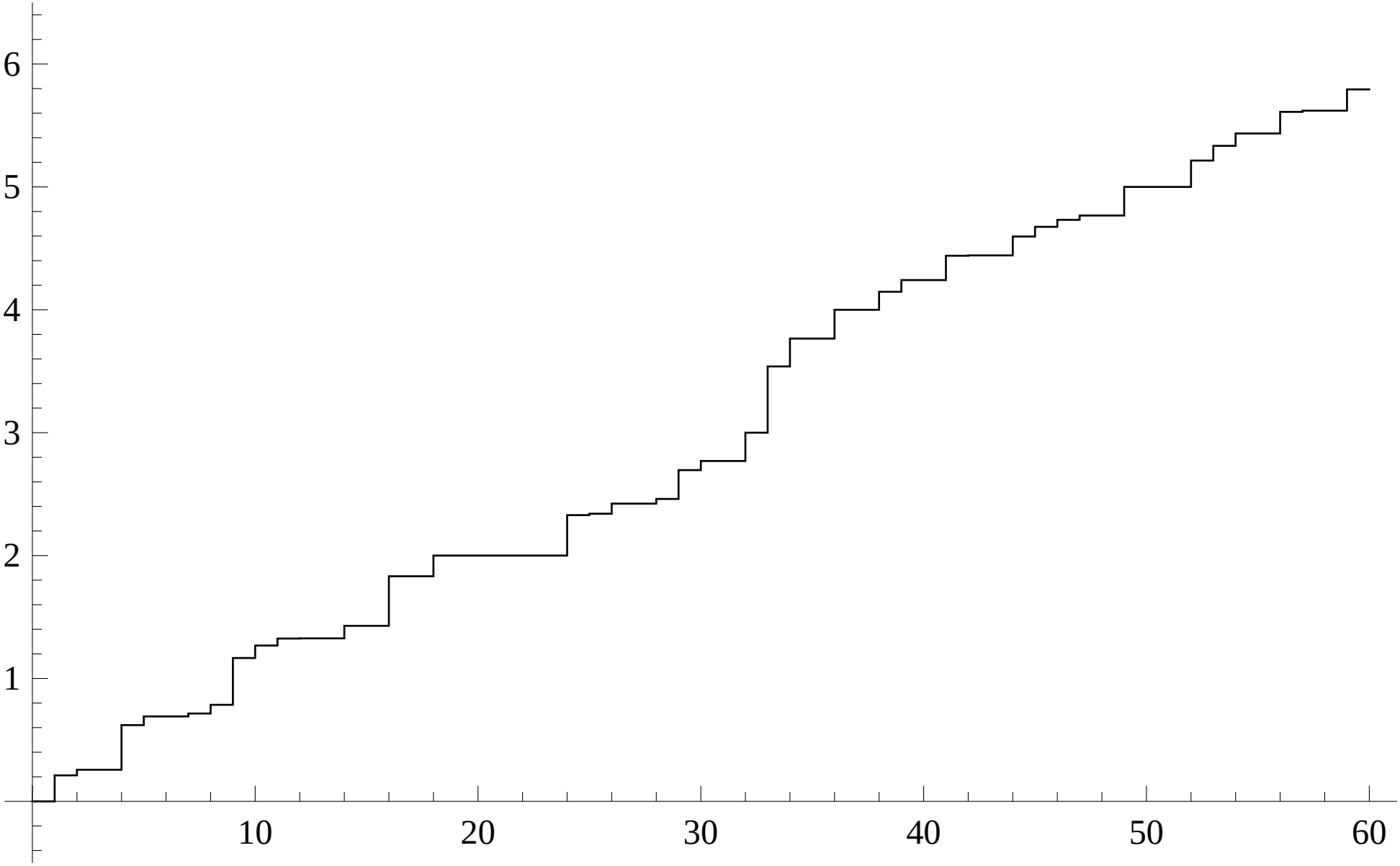
B3

$$P(x) = (-5+x)^3(-4+x)^2(-3+x)(-2+x)^6x$$
$$(6-6x+x^2)(-17+23x-9x^2+x^3)$$
$$(-13+23x-9x^2+x^3)(36-88x+56x^2-13x^3+x^4)$$
$$(-656+5640x-15880x^2+21396x^3-15978x^4+$$
$$7088x^5-1911x^6+307x^7-27x^8+x^9)$$
$$(-42800+402552x-1483372x^2+2978576x^3-$$
$$3734072x^4+3136628x^5-1836114x^6+764384x^7-$$
$$227689x^8+48165x^9-7062x^{10}+682x^{11}-39x^{12}+x^{13})^2$$

B3

0	(1)	2	(6)	4.43986	(1)
0.21166	(1)	2.32916	(1)	4.44251	(2)
0.257262	(2)	2.34075	(1)	4.59634	(1)
0.620488	(1)	2.42286	(2)	4.67513	(1)
0.691299	(2)	2.46081	(1)	4.73205	(1)
0.714929	(1)	2.69543	(1)	4.76701	(2)
0.78568	(1)	2.7697	(2)	5	(3)
1.16651	(1)	3	(1)	5.21432	(1)
1.26795	(1)	3.53919	(1)	5.33412	(1)
1.32487	(1)	3.76593	(2)	5.4348	(2)
1.32638	(2)	4	(2)	5.6105	(1)
1.4285	(2)	4.14692	(1)	5.62019	(2)
1.83172	(2)	4.24185	(2)	5.79334	(1)

B3

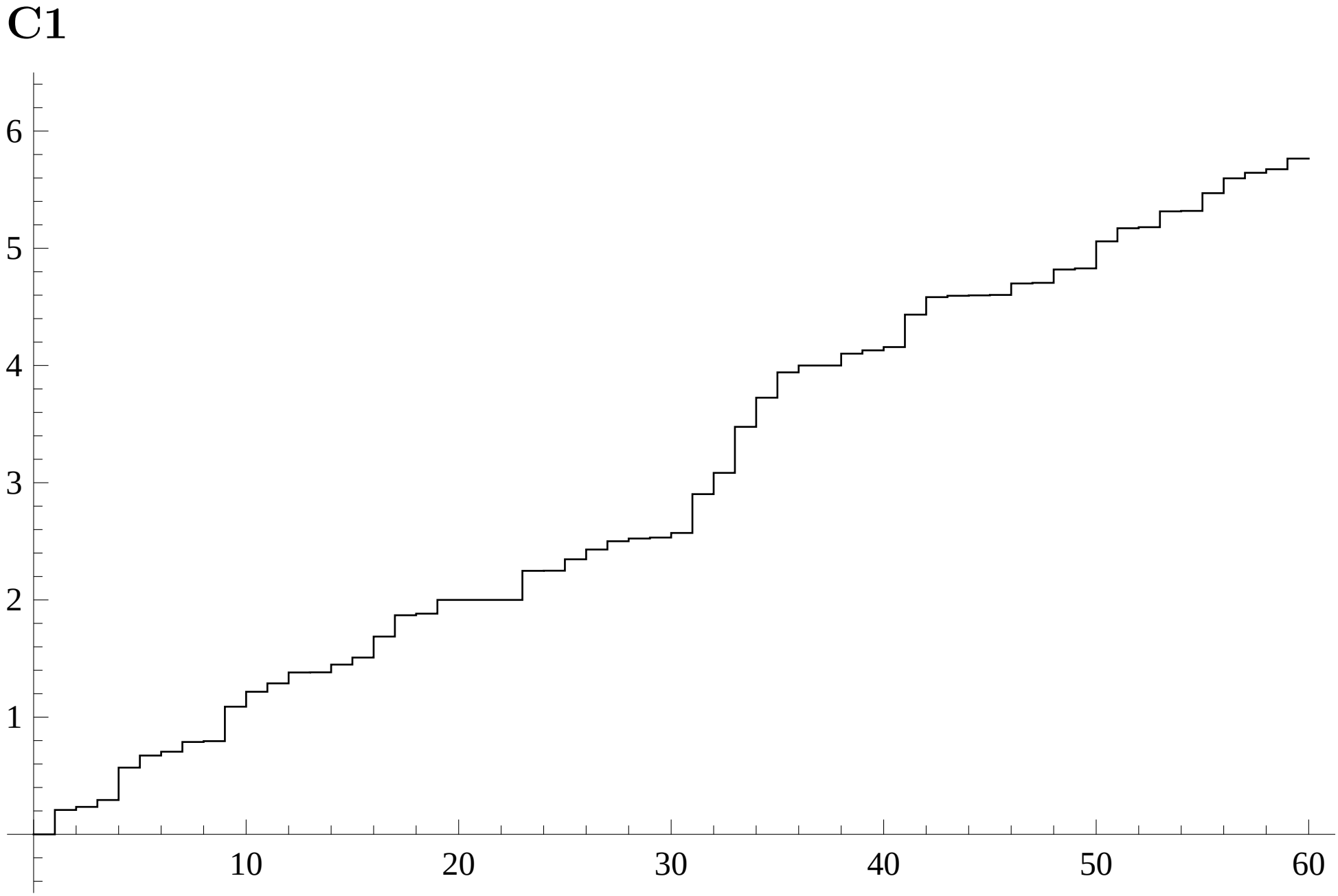


C1

$$P(x) = (-4+x)^2(-2+x)^4x$$
$$\left(-77190+383344x-808656x^2+968855x^3-738553x^4+378242x^5-\right.$$
$$\left.133356x^6+32475x^7-5368x^8+575x^9-36x^{10}+x^{11}\right)$$
$$\left(12796-127138x+459112x^2-868524x^3+996257x^4-746079x^5+\right.$$
$$\left.379464x^6-133464x^7+32479x^8-5368x^9+575x^{10}-36x^{11}+x^{12}\right)$$
$$\left(-708904+7249920x-29235644x^2+65418908x^3-93378986x^4+91499228x^5-\right.$$
$$64241260x^6+33151505x^7-12748827x^8+3669812x^9-787140x^{10}+123927x^{11}-$$
$$\left.13900x^{12}+1051x^{13}-48x^{14}+x^{15}\right)$$
$$\left(-122520+1325504x-6041028x^2+15619080x^3-25903278x^4+29440464x^5-\right.$$
$$23854840x^6+14113875x^7-6178453x^8+2009898x^9-483788x^{10}+84911x^{11}-$$
$$\left.10552x^{12}+879x^{13}-44x^{14}+x^{15}\right)$$

C1

0	(1)	1.44834	(1)	2.90263	(1)	4.69979	(1)
0.208075	(1)	1.50818	(1)	3.08386	(1)	4.70525	(1)
0.234302	(1)	1.68749	(1)	3.47667	(1)	4.81905	(1)
0.293047	(1)	1.86937	(1)	3.72537	(1)	4.82841	(1)
0.569279	(1)	1.88285	(1)	3.94156	(1)	5.05902	(1)
0.672341	(1)	2	(4)	4	(2)	5.17103	(1)
0.70499	(1)	2.24815	(1)	4.1011	(1)	5.18004	(1)
0.788051	(1)	2.24935	(1)	4.12931	(1)	5.31501	(1)
0.795289	(1)	2.34651	(1)	4.15729	(1)	5.31908	(1)
1.08927	(1)	2.43012	(1)	4.43417	(1)	5.47038	(1)
1.21657	(1)	2.50084	(1)	4.58329	(1)	5.59677	(1)
1.28822	(1)	2.52402	(1)	4.59482	(1)	5.64418	(1)
1.3812	(1)	2.53204	(1)	4.5984	(1)	5.67451	(1)
1.38239	(1)	2.57144	(1)	4.602	(1)	5.76529	(1)


C1
6
5
4
3
2
1
10
20
30
40
50
60

C2

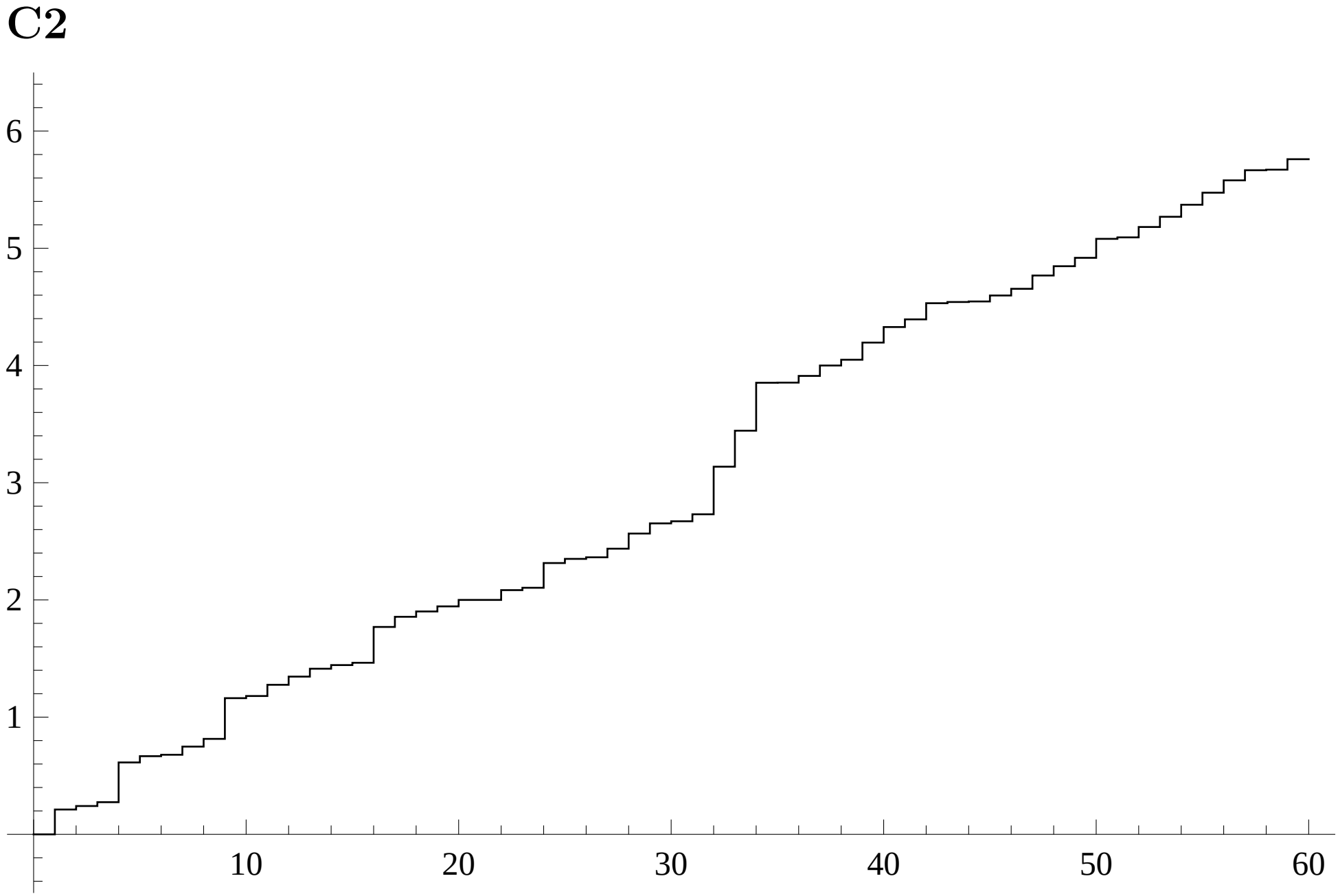
$$P(x) = (-4+x)(-2+x)^2 x$$

$$\left(-37428459090 + 604203411508x - 4439605146812x^2 + 19930998515948x^3 - 61755029854010x^4 + 141222575037312x^5 - 248653284601630x^6 + 346956842494078x^7 - 391672536112820x^8 + 363206410862681x^9 - 279840678487636x^{10} + 180669097360862x^{11} - 98348275174002x^{12} + 45333871516174x^{13} - 17741176121806x^{14} + 5900531116855x^{15} - 1666923037100x^{16} + 399123554592x^{17} - 80671217476x^{18} + 13678974433x^{19} - 1928531216x^{20} + 223239508x^{21} - 20844212x^{22} + 1530331x^{23} - 85016x^{24} + 3358x^{25} - 84x^{26} + x^{27}\right)$$

$$\left(-36746254304 + 772648677272x - 7242350092638x^2 + 40672228102548x^3 - 155269561221928x^4 + 432744830058116x^5 - 921795998163530x^6 + 1548970088408384x^7 - 2100872609075270x^8 + 2339432022014066x^9 - 2166639572400320x^{10} + 1685419746967525x^{11} - 1109483320904948x^{12} + 621489468253834x^{13} - 297404593302438x^{14} + 121880797787010x^{15} - 42823782063958x^{16} + 12897698021387x^{17} - 3324599103268x^{18} + 731208617716x^{19} - 136567671024x^{20} + 21512205773x^{21} - 2830902584x^{22} + 307177364x^{23} - 26989232x^{24} + 1871027x^{25} - 98456x^{26} + 3694x^{27} - 88x^{28} + x^{29}\right)$$

C2

0	(1)	1.46391	(1)	2.73079	(1)	4.65432	(1)
0.211929	(1)	1.76952	(1)	3.13663	(1)	4.76788	(1)
0.241693	(1)	1.8562	(1)	3.44386	(1)	4.84713	(1)
0.274516	(1)	1.90158	(1)	3.85282	(1)	4.91869	(1)
0.613566	(1)	1.94504	(1)	3.85398	(1)	5.08058	(1)
0.667022	(1)	2	(2)	3.91119	(1)	5.09311	(1)
0.679086	(1)	2.08368	(1)	4	(1)	5.18174	(1)
0.748687	(1)	2.10372	(1)	4.04915	(1)	5.26882	(1)
0.814617	(1)	2.3147	(1)	4.19539	(1)	5.37156	(1)
1.16193	(1)	2.35019	(1)	4.32815	(1)	5.47419	(1)
1.18024	(1)	2.36363	(1)	4.39369	(1)	5.57963	(1)
1.27602	(1)	2.4372	(1)	4.53148	(1)	5.66609	(1)
1.34586	(1)	2.56642	(1)	4.54172	(1)	5.67096	(1)
1.41304	(1)	2.65313	(1)	4.54635	(1)	5.76011	(1)
1.44418	(1)	2.6713	(1)	4.59735	(1)		


C2
6
5
4
3
2
1
10
20
30
40
50
60

C3

$$P(x) = (-4+x)^4(-2+x)^6x$$

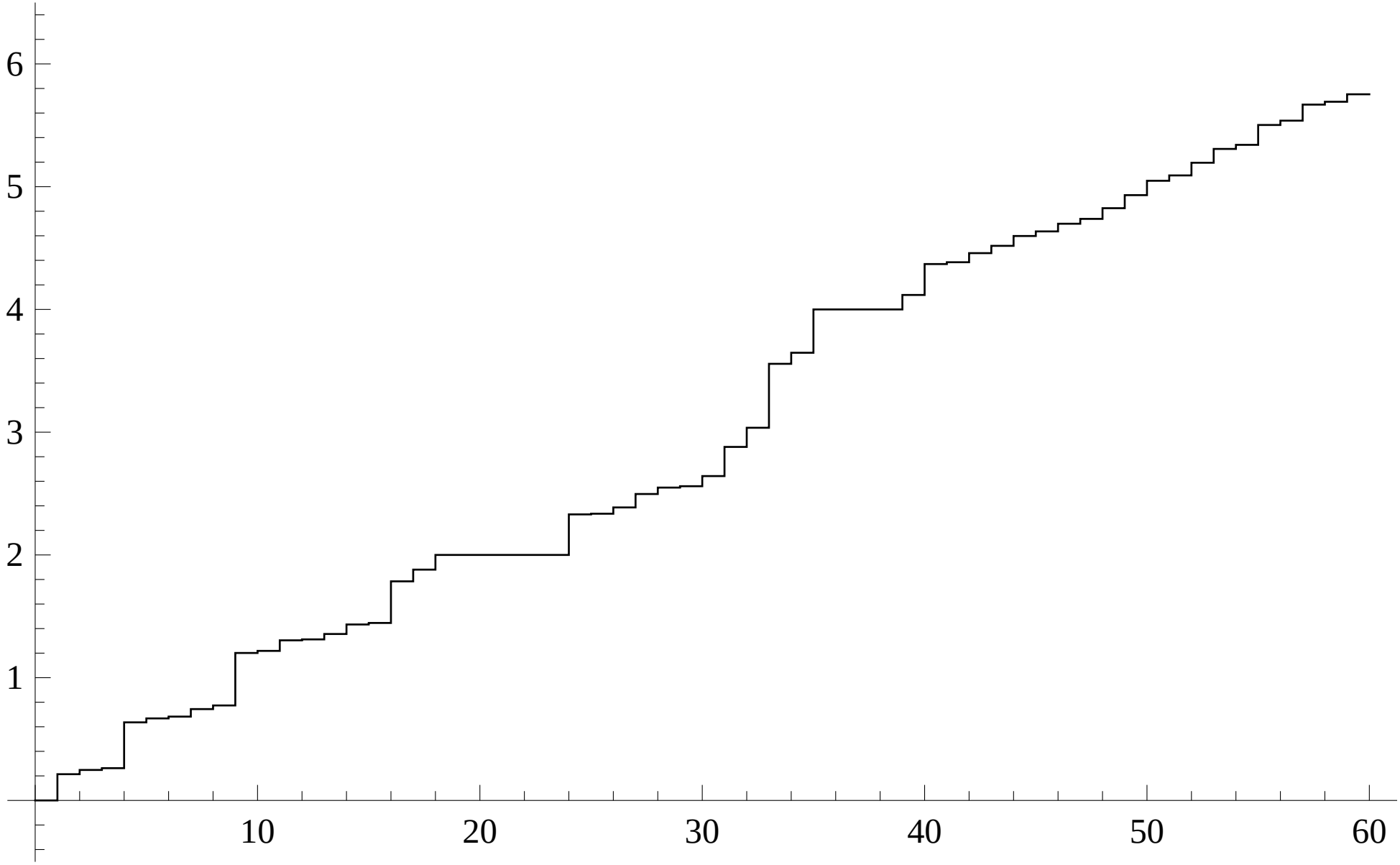
$$\left(-557660340 + 8329233813x - 55127691398x^2 + 218090879535x^3 - 583814567550x^4 + 1131872007137x^5 - 1658340411710x^6 + 1889549591178x^7 - 1708486233852x^8 + 1243674906498x^9 - 736388512220x^{10} + 357152561775x^{11} - 142491170544x^{12} + 46843843732x^{13} - 12679508042x^{14} + 2815931075x^{15} - 509718844x^{16} + 74409012x^{17} - 8620944x^{18} + 773907x^{19} - 51872x^{20} + 2442x^{21} - 72x^{22} + x^{23}\right)$$

$$\left(2410005600 - 46931818200x + 403273714784x^2 - 2054033992636x^3 + 7034425363933x^4 - 17393056574982x^5 + 32499548833503x^6 - 47357574728470x^7 + 55046736592289x^8 - 51897017413050x^9 + 40180314421430x^{10} - 25783308356500x^{11} + 13803794421854x^{12} - 6193483471180x^{13} + 2334821012579x^{14} - 740058368400x^{15} + 197006379064x^{16} - 43907214310x^{17} + 8148340939x^{18} - 1248770268x^{19} + 156156668x^{20} - 15661144x^{21} + 1228523x^{22} - 72568x^{23} + 3034x^{24} - 80x^{25} + x^{26}\right)$$

C3

0	(1)	1.43345	(1)	3.55698	(1)	5.04818	(1)
0.213782	(1)	1.4459	(1)	3.64708	(1)	5.09176	(1)
0.24838	(1)	1.78491	(1)	4	(4)	5.19503	(1)
0.262937	(1)	1.88043	(1)	4.11791	(1)	5.30765	(1)
0.636157	(1)	2	(6)	4.3697	(1)	5.34084	(1)
0.668311	(1)	2.33032	(1)	4.38454	(1)	5.50272	(1)
0.682986	(1)	2.33582	(1)	4.45869	(1)	5.53788	(1)
0.744163	(1)	2.38699	(1)	4.51807	(1)	5.66859	(1)
0.773784	(1)	2.49638	(1)	4.59825	(1)	5.69194	(1)
1.20139	(1)	2.54869	(1)	4.63568	(1)	5.75281	(1)
1.21837	(1)	2.55986	(1)	4.69816	(1)		
1.30437	(1)	2.64241	(1)	4.73772	(1)		
1.31199	(1)	2.88001	(1)	4.82485	(1)		
1.35598	(1)	3.03608	(1)	4.93113	(1)		

C3



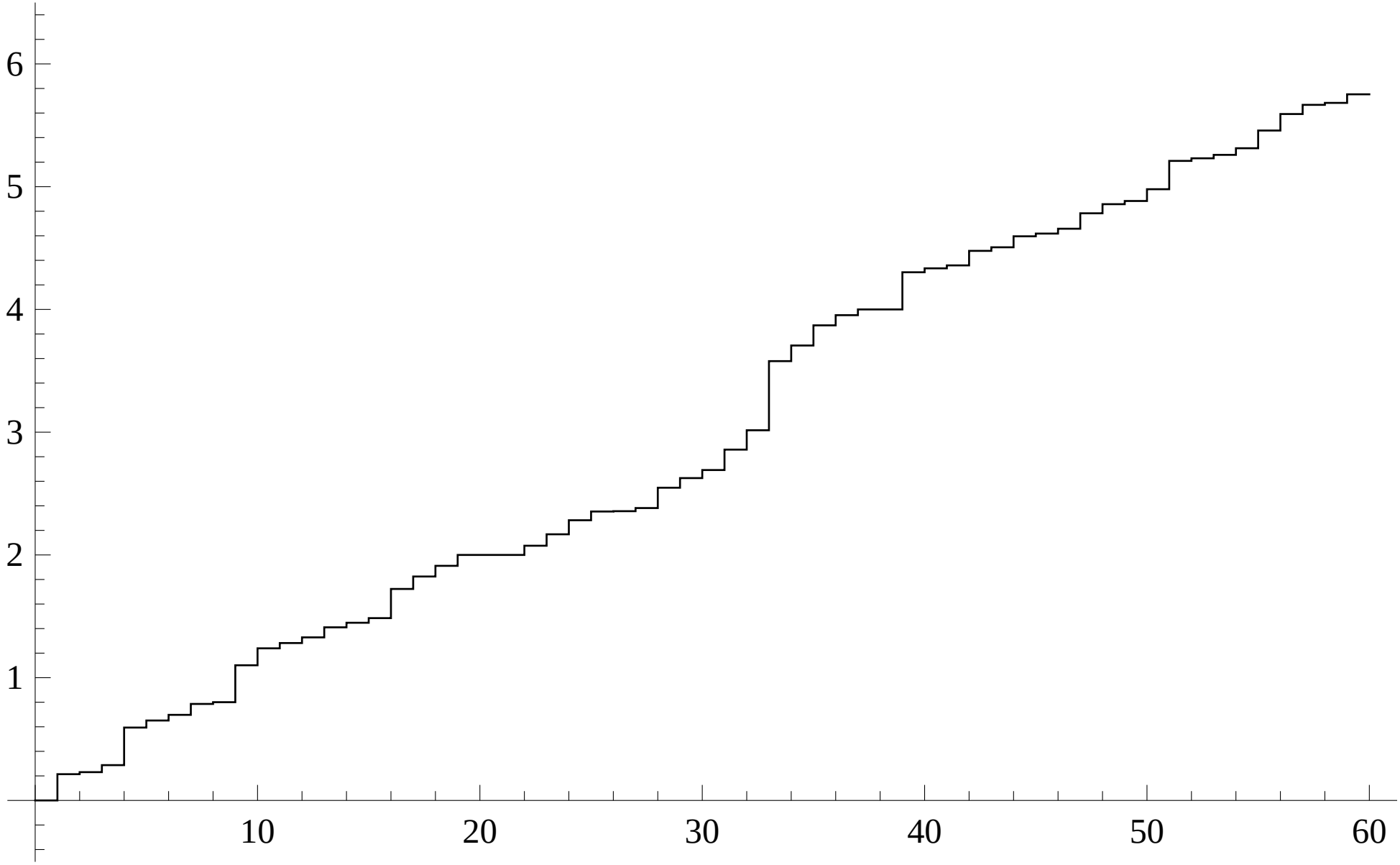
C4

$$P(x) = (-4+x)^2(-2+x)^3x\left(11-7x+x^2\right)\left(3-5x+x^2\right)$$
$$\left(-129023280+1762939170x-10373152354x^2+35856778041x^3-\right.$$
$$82868029210x^4+137412738354x^5-170803393706x^6+163832764158x^7-$$
$$123702404356x^8+74535044543x^9-36163118046x^{10}+14202908986x^{11}-$$
$$4523997070x^{12}+1167075464x^{13}-242634794x^{14}+40278230x^{15}-$$
$$5259974x^{16}+528041x^{17}-39298x^{18}+2041x^{19}-66x^{20}+\left.x^{21}\right)$$
$$\left(-40345012608+825626945888x-7581731395544x^2+419676268101 12x^3-\right.$$
$$158625272471604x^4+4390765835698 46x^5-9308862962614 60x^6+$$
$$155919206136430 3x^7-2110057937496018x^8+2346121076896025x^9-$$
$$2170625985791100x^{10}+1687376498674081x^{11}-1110277288022374x^{12}+$$
$$621756071173331x^{13}-297478575478422x^{14}+121897695878691x^{15}-$$
$$42826936324474x^{16}+12898173909029x^{17}-3324656188818x^{18}+$$
$$731213931104x^{19}-136568040684x^{20}+21512223851x^{21}-$$
$$2830903138x^{22}+307177372x^{23}-26989232x^{24}+$$
$$\left.1871027x^{25}-98456x^{26}+3694x^{27}-88x^{28}+x^{29}\right)$$

C4

0	(1)	1.4852	(1)	3.01578	(1)	4.85754	(1)
0.213932	(1)	1.72292	(1)	3.57886	(1)	4.88352	(1)
0.230532	(1)	1.8243	(1)	3.70613	(1)	4.97943	(1)
0.287644	(1)	1.91131	(1)	3.87036	(1)	5.21003	(1)
0.593695	(1)	2	(3)	3.9534	(1)	5.23129	(1)
0.651185	(1)	2.07507	(1)	4	(2)	5.25932	(1)
0.697224	(1)	2.16824	(1)	4.30278	(1)	5.31323	(1)
0.786133	(1)	2.28271	(1)	4.33484	(1)	5.45746	(1)
0.800498	(1)	2.35383	(1)	4.35882	(1)	5.59193	(1)
1.10127	(1)	2.35652	(1)	4.47718	(1)	5.66668	(1)
1.23918	(1)	2.38197	(1)	4.5063	(1)	5.68267	(1)
1.28228	(1)	2.54758	(1)	4.59582	(1)	5.75266	(1)
1.32842	(1)	2.62598	(1)	4.61803	(1)		
1.41032	(1)	2.69164	(1)	4.65758	(1)		
1.44752	(1)	2.85805	(1)	4.78326	(1)		

C4



C5

$$P(x) = (-4+x)(-2+x)^3 x$$

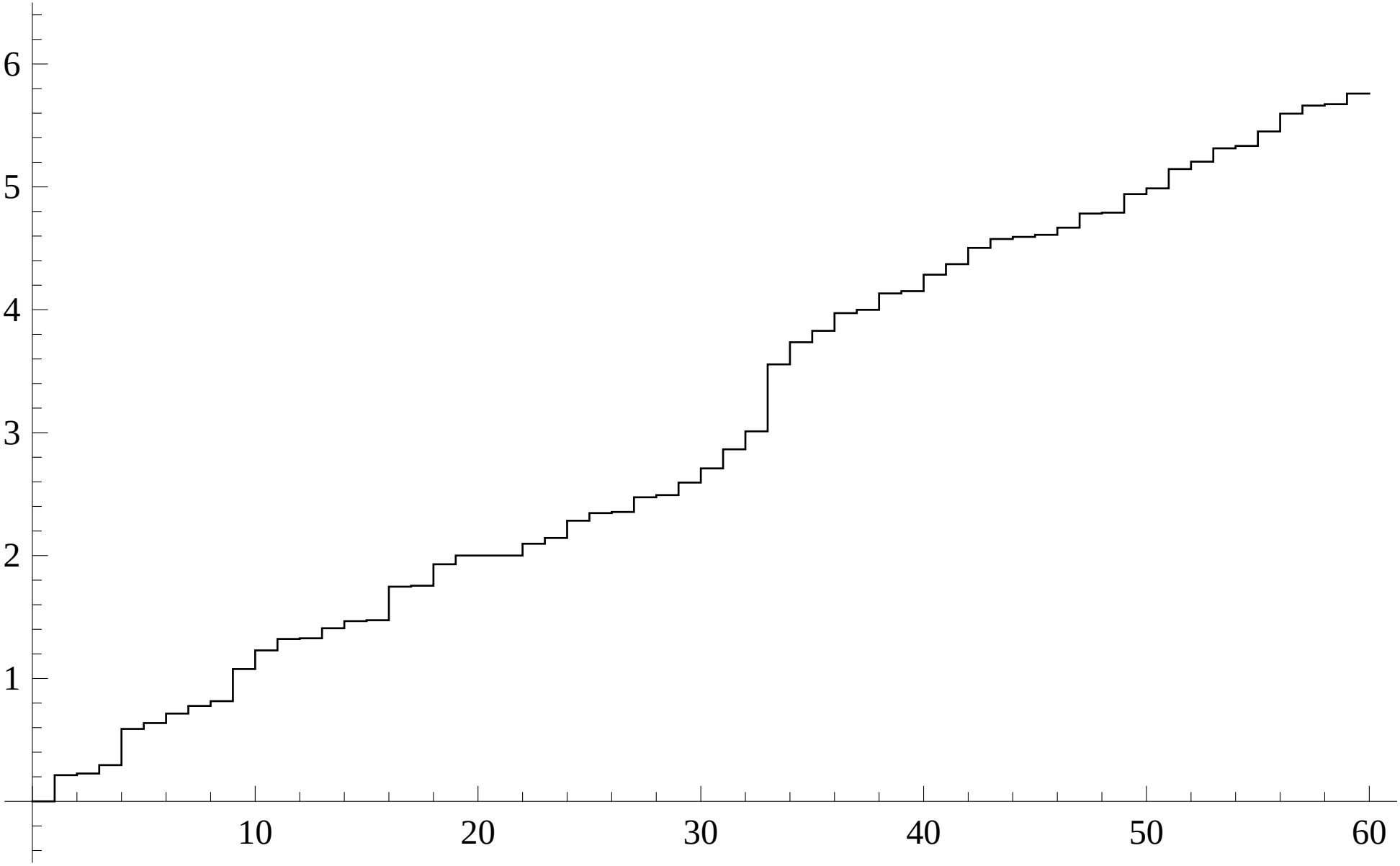
$$\left(-33922169760 + 568756091408x - 4273869055244x^2 + 19453794780368x^3 - 60804681647648x^4 + 139829058004980x^5 - 247089900601620x^6 + 345580569056844x^7 - 390705471315120x^8 + 362657726468648x^9 - 279587461514156x^{10} + 180573673073441x^{11} - 98318887715262x^{12} + 45326492595349x^{13} - 17739674184940x^{14} + 5900285743270x^{15} - 1666891356182x^{16} + 399120397310x^{17} - 80670983362x^{18} + 13678962279x^{19} - 1928530822x^{20} + 223239502x^{21} - 20844212x^{22} + 1530331x^{23} - 85016x^{24} + 3358x^{25} - 84x^{26} + x^{27}\right)$$

$$\left(20230695168 - 403204324480x + 35859749554 4x^2 - 1917803541974 8x^3 + 6966244229985 6x^4 - 1845302851235 84x^5 + 3728387352878 52x^6 - 5927033887709 24x^7 + 7581741538284 44x^8 - 7935540143582 24x^9 + 6882566078735 24x^{10} - 4994094586068 30x^{11} + 3053678614333 85x^{12} - 1581703856573 84x^{13} + 6964712155356 9x^{14} - 2612361271256 4x^{15} + 8351335336618x^{16} - 2273368119446x^{17} + 525637688246x^{18} - 102787513060x^{19} + 16890220371x^{20} - 2310999566x^{21} + 259951718x^{22} - 23612826x^{23} + 1688203x^{24} - 91412x^{25} + 3522x^{26} - 86x^{27} + x^{28}\right)$$

C5

0	(1)	1.47383	(1)	3.01108	(1)	4.78297	(1)
0.213292	(1)	1.74694	(1)	3.556	(1)	4.79053	(1)
0.226571	(1)	1.75479	(1)	3.73591	(1)	4.94112	(1)
0.294996	(1)	1.92938	(1)	3.82893	(1)	4.9879	(1)
0.589475	(1)	2	(3)	3.97304	(1)	5.14521	(1)
0.637226	(1)	2.09658	(1)	4	(1)	5.20526	(1)
0.714477	(1)	2.14346	(1)	4.13326	(1)	5.31387	(1)
0.776437	(1)	2.28383	(1)	4.15151	(1)	5.33335	(1)
0.815448	(1)	2.34572	(1)	4.2857	(1)	5.45093	(1)
1.07678	(1)	2.3549	(1)	4.37152	(1)	5.59591	(1)
1.22836	(1)	2.47419	(1)	4.50397	(1)	5.66156	(1)
1.32137	(1)	2.49212	(1)	4.57605	(1)	5.67328	(1)
1.32688	(1)	2.59407	(1)	4.59323	(1)	5.75932	(1)
1.40851	(1)	2.70923	(1)	4.61004	(1)		
1.46648	(1)	2.86474	(1)	4.66844	(1)		

C5



C6

$$P(x) = (-4+x)(-2+x)^2 x$$

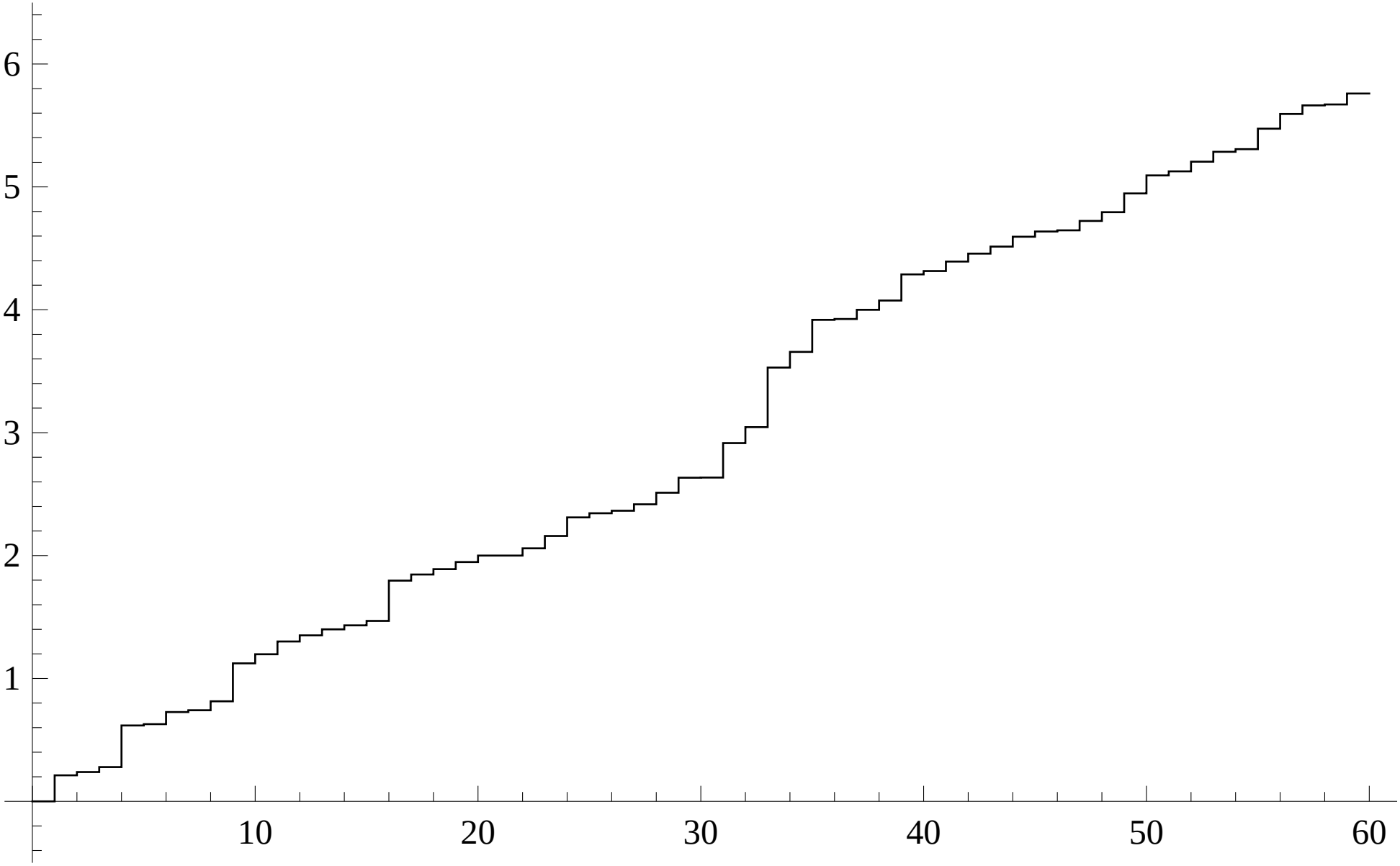
$$\left(-34967090610 + 579606010332x - 4325898016546x^2 + 19607032184489x^3 - 61115828159474x^4 + 140292645192410x^5 - 247616624013482x^6 + 346048727402660x^7 - 391036704281824x^8 + 362846510882591x^9 - 279674805272550x^{10} + 180606616413596x^{11} - 98329027861608x^{12} + 45329034539346x^{13} - 17740190280910x^{14} + 5900369787644x^{15} - 1666902166646x^{16} + 399121470172x^{17} - 80671062558x^{18} + 13678966371x^{19} - 1928530954x^{20} + 223239504x^{21} - 20844212x^{22} + 1530331x^{23} - 85016x^{24} + 3358x^{25} - 84x^{26} + x^{27}\right)$$

$$\left(-39316085216 + 809869928856x - 7475320160542x^2 + 41538241498812x^3 - 157451952596022x^4 + 436751531963945x^5 - 927398696403190x^6 + 1555115669940542x^7 - 2106268988812494x^8 + 2343279494846596x^9 - 2168888777345920x^{10} + 1686504751944827x^{11} - 1109916755167630x^{12} + 621632996272980x^{13} - 297443932147796x^{14} + 121889685089750x^{15} - 42825425023798x^{16} + 12897943795880x^{17} - 3324628366186x^{18} + 731211323744x^{19} - 136567858222x^{20} + 21512214883x^{21} - 2830902862x^{22} + 307177368x^{23} - 26989232x^{24} + 1871027x^{25} - 98456x^{26} + 3694x^{27} - 88x^{28} + x^{29}\right)$$

C6

0	(1)	1.4684	(1)	2.91517	(1)	4.64654	(1)
0.211974	(1)	1.79611	(1)	3.04517	(1)	4.72305	(1)
0.238374	(1)	1.84607	(1)	3.52962	(1)	4.79433	(1)
0.278953	(1)	1.88955	(1)	3.65748	(1)	4.94683	(1)
0.617458	(1)	1.94745	(1)	3.91813	(1)	5.09342	(1)
0.628393	(1)	2	(2)	3.9251	(1)	5.12653	(1)
0.726758	(1)	2.05898	(1)	4	(1)	5.20525	(1)
0.741417	(1)	2.15954	(1)	4.07535	(1)	5.28568	(1)
0.81431	(1)	2.31076	(1)	4.28812	(1)	5.30702	(1)
1.12288	(1)	2.34447	(1)	4.31515	(1)	5.47368	(1)
1.19731	(1)	2.36496	(1)	4.39262	(1)	5.59355	(1)
1.30118	(1)	2.41717	(1)	4.45718	(1)	5.66317	(1)
1.35071	(1)	2.51149	(1)	4.51427	(1)	5.67096	(1)
1.39981	(1)	2.63354	(1)	4.59456	(1)	5.75995	(1)
1.43222	(1)	2.63488	(1)	4.63699	(1)		

C6



C7

$$P(x) = (-4+x)(-2+x)^2 x$$

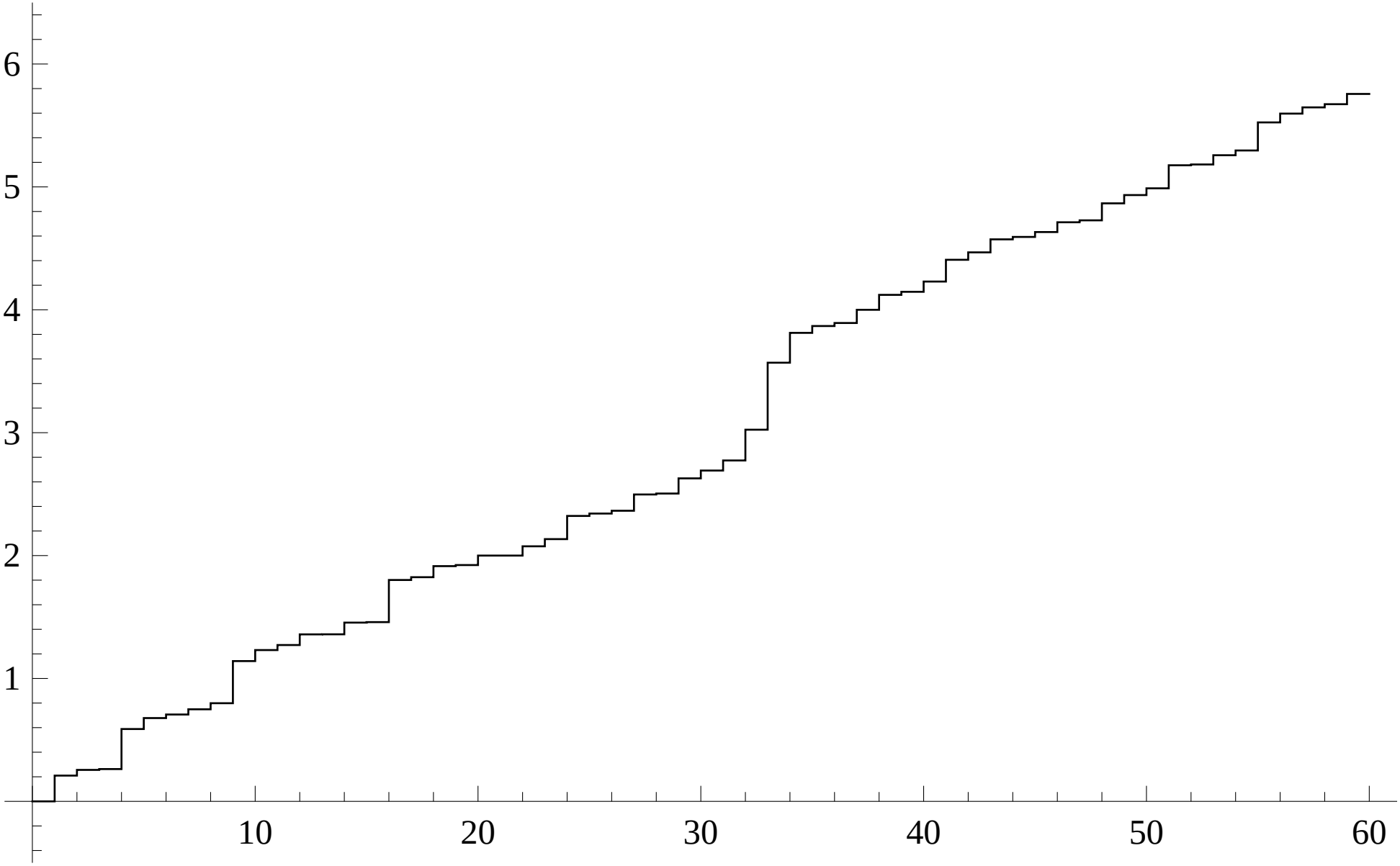
$$\left(-36261801000 + 591286905480x - 4374287654760x^2 + 19729719074656x^3 - 61329846333828x^4 + 140566290646385x^5 - 247883256486918x^6 + 346251830818283x^7 - 391159745323658x^8 + 362906464888161x^9 - 279698462809438x^{10} + 180614198398778x^{11} - 98331000102176x^{12} + 45329449056470x^{13} - 17740260045104x^{14} + 5900379052075x^{15} - 1666903115252x^{16} + 399121542392x^{17} - 80671066406x^{18} + 13678966499x^{19} - 1928530956x^{20} + 223239504x^{21} - 20844212x^{22} + 1530331x^{23} - 85016x^{24} + 3358x^{25} - 84x^{26} + x^{27}\right)$$

$$\left(-37977713664 + 791854091040x - 7371991048648x^2 + 41189656672680x^3 - 156660826133392x^4 + 435452008477136x^5 - 925782305873736x^6 + 1553546947827081x^7 - 2105056326018858x^8 + 2342522043588459x^9 - 2168502745713150x^{10} + 1686343240922601x^{11} - 1109861113825538x^{12} + 621617211176454x^{13} - 297440255184392x^{14} + 121888986211650x^{15} - 42825317756412x^{16} + 12897930710999x^{17} - 3324627127756x^{18} + 731211236116x^{19} - 136567853862x^{20} + 21512214747x^{21} - 2830902860x^{22} + 307177368x^{23} - 26989232x^{24} + 1871027x^{25} - 98456x^{26} + 3694x^{27} - 88x^{28} + x^{29}\right)$$

C7

0	(1)	1.45861	(1)	2.77388	(1)	4.71221	(1)
0.209767	(1)	1.8014	(1)	3.02428	(1)	4.72791	(1)
0.256049	(1)	1.82409	(1)	3.56947	(1)	4.86619	(1)
0.262854	(1)	1.91414	(1)	3.81224	(1)	4.93357	(1)
0.588647	(1)	1.92288	(1)	3.86789	(1)	4.98844	(1)
0.677806	(1)	2	(2)	3.89251	(1)	5.17602	(1)
0.706796	(1)	2.07553	(1)	4	(1)	5.18254	(1)
0.749249	(1)	2.13399	(1)	4.12149	(1)	5.25773	(1)
0.79868	(1)	2.32257	(1)	4.14649	(1)	5.29646	(1)
1.14153	(1)	2.34202	(1)	4.22988	(1)	5.52479	(1)
1.23117	(1)	2.36475	(1)	4.40696	(1)	5.59641	(1)
1.27216	(1)	2.4972	(1)	4.46731	(1)	5.64712	(1)
1.35888	(1)	2.5049	(1)	4.57318	(1)	5.67302	(1)
1.35948	(1)	2.62824	(1)	4.59292	(1)	5.75643	(1)
1.4546	(1)	2.69197	(1)	4.6327	(1)		

C7



C8

$$P(x) = (-4 + x)(-2 + x)^3 x$$

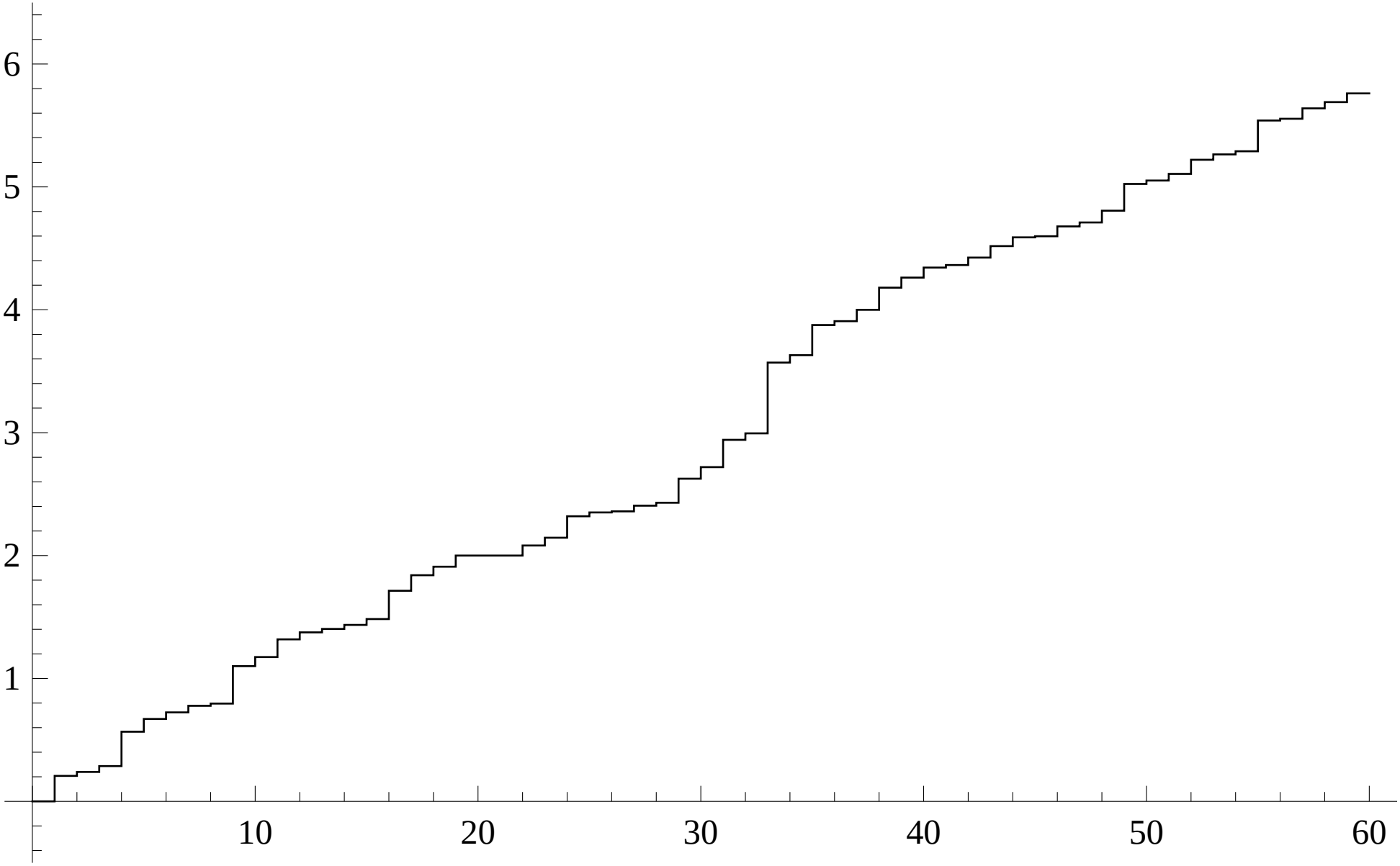
$$\left(-37246511880 + 600024377592x - 4409642491698x^2 + 19816704103799x^3 - 61476159405824x^4 + 140745521686511x^5 - 248049445566262x^6 + 346371423937785x^7 - 391227639455864x^8 + 362937185565167x^9 - 279709601981724x^{10} + 180617438775843x^{11} - 98331754000156x^{12} + 45329588241609x^{13} - 17740280155706x^{14} + 5900381276062x^{15} - 1666903296900x^{16} + 399121552712x^{17} - 80671066770x^{18} + 13678966505x^{19} - 1928530956x^{20} + 223239504x^{21} - 20844212x^{22} + 1530331x^{23} - 85016x^{24} + 3358x^{25} - 84x^{26} + x^{27} \right)$$

$$\left(18438568960 - 379452471680x + 3455874909952x^2 - 18735282798356x^3 + 68675450103107x^4 - 182935985726254x^5 + 370886164113495x^6 - 590835132243188x^7 + 756748622300193x^8 - 792674204737282x^9 + 687813182247715x^{10} - 499225874728386x^{11} + 305305252809095x^{12} - 158152800016230x^{13} + 696430659819 69x^{14} - 261228497358 14x^{15} + 83512194718 94x^{16} - 22733541429 16x^{17} + 5256363809 32x^{18} - 1027874217 04x^{19} + 168902158 85x^{20} - 23109994 28x^{21} + 2599517 16x^{22} - 236128 26x^{23} + 16882 03x^{24} - 914 12x^{25} + 35 22x^{26} - 86x^{27} + x^{28} \right)$$

C8

0	(1)	1.4834	(1)	2.99457	(1)	4.71063	(1)
0.207301	(1)	1.71369	(1)	3.57028	(1)	4.80626	(1)
0.239529	(1)	1.84019	(1)	3.63061	(1)	5.02422	(1)
0.287013	(1)	1.90961	(1)	3.8758	(1)	5.05176	(1)
0.566741	(1)	2	(3)	3.9074	(1)	5.10603	(1)
0.67048	(1)	2.0819	(1)	4	(1)	5.22088	(1)
0.724009	(1)	2.1454	(1)	4.18006	(1)	5.26439	(1)
0.777687	(1)	2.31995	(1)	4.26187	(1)	5.28993	(1)
0.795785	(1)	2.35093	(1)	4.34344	(1)	5.54023	(1)
1.10029	(1)	2.35987	(1)	4.36417	(1)	5.55449	(1)
1.17426	(1)	2.40602	(1)	4.42478	(1)	5.63897	(1)
1.31837	(1)	2.43014	(1)	4.51821	(1)	5.69004	(1)
1.37515	(1)	2.62553	(1)	4.58938	(1)	5.76087	(1)
1.4031	(1)	2.7197	(1)	4.59849	(1)		
1.43587	(1)	2.94188	(1)	4.67845	(1)		

C8



Discrete heat kernel

$$H(t) = \exp(-tA)$$

Green matrix

$$G(a) = (aI + A)^{-1} = \int_0^\infty e^{-at} H(t) dt$$

Pseudo Green matrix

$$G_* = \lim_{a \to +0} \left(G(a) - \frac{1}{a} E_0 \right), \qquad E_0 = \frac{1}{60} 1^t 1$$

$$1 = {}^t(1, \cdots, 1)$$

Sobolev energy

$$\boldsymbol{E(u) = \sum_{(i,j) \in e} |u(i) - u(j)|^2 = u^* A u}$$

$$\boldsymbol{E(a,u) = E(u) + a \sum_{j=0}^{59} |u(j)|^2 = u^*(A + aI)u}$$

$$\boldsymbol{u = {}^t(u(0), u(1), \cdots, u(59)) \in \mathrm{C}^{60}, \qquad 0 < a < \infty}$$

Theorem 1

$$u \in \mathrm{C}^{60} \quad \text{and} \quad u(0) + u(1) + \cdots + u(59) = 0 \quad \Longrightarrow$$

$$\left(\max_{0 \leq j \leq 59} | u(j) | \right)^2 \leq C\, E(u)$$

$$C_0 = \max_{0 \leq j \leq 59} {}^t\delta_j G_* \delta_j = {}^t\delta_{j_0} G_* \delta_{j_0}$$

If we replace C by C_0 then the equality holds for j_0-th column vector of G_*.

$$\delta_j = {}^t(0, \cdots, 0, \overbrace{1}^{j}, 0, \cdots, 0)$$

$$C_0(\mathrm{A1}) = \frac{1}{60}\sum_{k=1}^{59}\frac{1}{\lambda_k} = \frac{239741}{376200}$$

$$C_0(\mathrm{A2}) = \frac{36409091911}{55355731200}$$

$$C_0(\mathrm{B1}) = \frac{49616123}{74390400}$$

$$C_0(\mathrm{B2}) = \frac{25524226539887}{38264600989440}$$

$$C_0(\mathrm{B3}) = \frac{3160823}{4737960}$$

$$C_0(\mathrm{C1}) = \frac{64245195133531571}{9574622890168700}$$

$$C_0(\mathrm{C2}) = \frac{34563382848227089229 53}{5157583784730001587600}$$

$$C_0(\mathrm{C3}) = \frac{156400481511242}{233327177482275}$$

$$C_0(\mathrm{C4}) = \frac{2469598657842821}{3681835419340320}$$

$$C_0(\mathrm{C5}) = \frac{899130319793743730 3}{1340369288772866640 0}$$

$$C_0(\mathrm{C6}) = \frac{384427839049445420497}{57282046424098059240 0}$$

$$C_0(\mathrm{C7}) = \frac{96432107634623811 7}{1434521140957238400}$$

$$C_0(\mathrm{C8}) = \frac{501412110751795 13}{7451957224596780 0}$$

$C_0(\mathrm{A1}) \doteqdot 0.63727$	0
$C_0(\mathrm{A2}) \doteqdot 0.657729$	0.0204593
$C_0(\mathrm{B1}) \doteqdot 0.666969$	0.0296994
$C_0(\mathrm{B2}) \doteqdot 0.667045$	0.0297753
$C_0(\mathrm{B3}) \doteqdot 0.667127$	0.0298573
$C_0(\mathrm{C1}) \doteqdot 0.670995$	0.0337245
$C_0(\mathrm{C2}) \doteqdot 0.670147$	0.0328767
$C_0(\mathrm{C3}) \doteqdot 0.670305$	0.0330354
$C_0(\mathrm{C4}) \doteqdot 0.670752$	0.033482
$C_0(\mathrm{C5}) \doteqdot 0.670808$	0.0335378
$C_0(\mathrm{C6}) \doteqdot 0.671114$	0.0338439
$C_0(\mathrm{C7}) \doteqdot 0.672225$	0.034955
$C_0(\mathrm{C8}) \doteqdot 0.67286$	0.0355896

Theorem 2

$u \in \mathrm{C}^{60} \implies$

$$\left(\max_{0 \le j \le 59} | u(j) | \right)^2 \le C\, E(a,u)$$

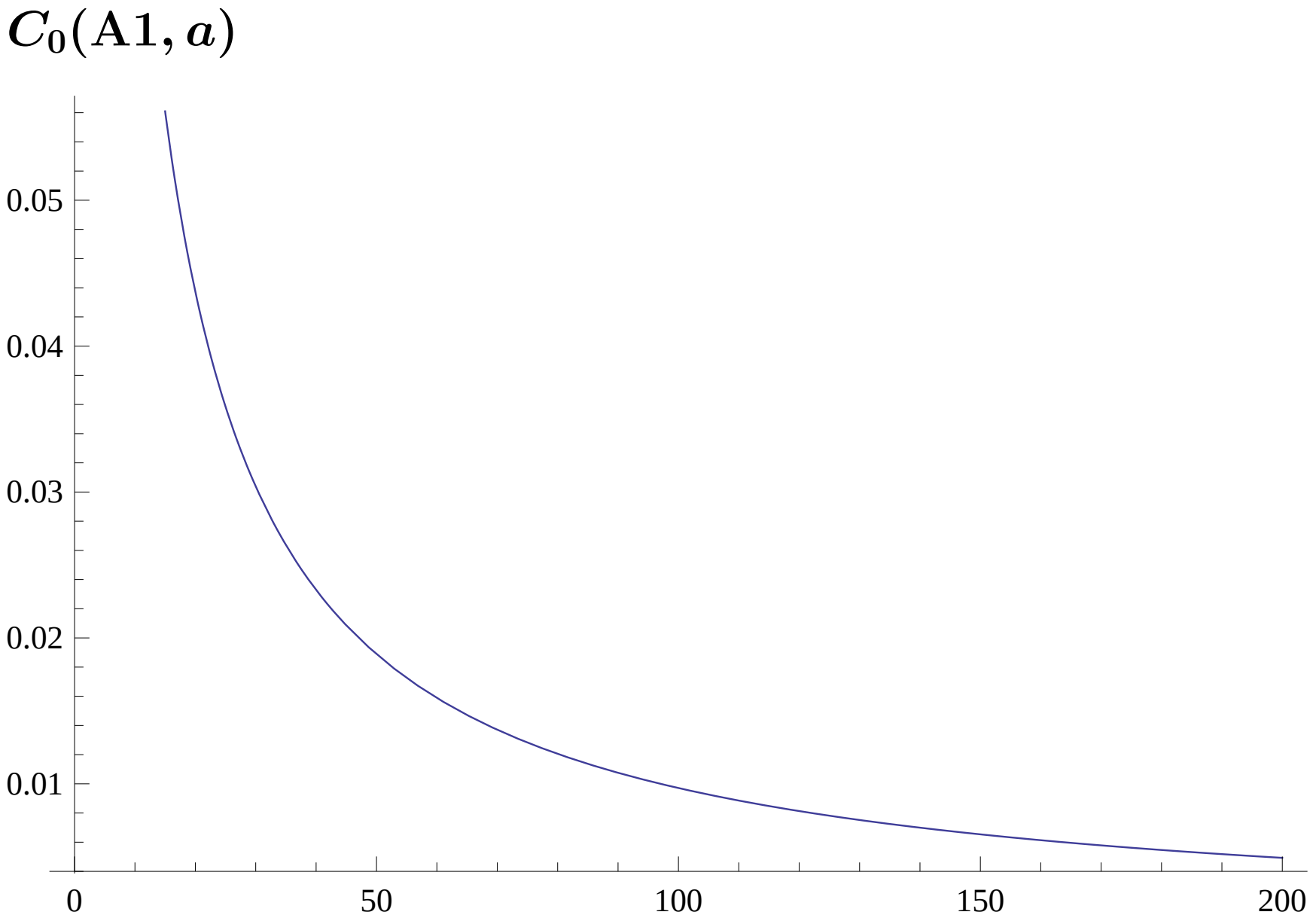
$$C_0(a) = \max_{0 \le j \le 59} {}^t\delta_j G(a) \delta_j = {}^t\delta_{j_0} G(a) \delta_{j_0}$$

If we replace C by $C_0(a)$ then the equality holds for j_0-th column vector of $G(a)$.

$$C_0(\mathrm{A1}, a) = \frac{1}{60} \sum_{k\,=\,0}^{59} \frac{1}{\lambda_k + a} = \frac{N(a)}{D(a)}$$

$$N(a) = 3344 + 160806a + 1153562a^2 + 3594661a^3 + 6334271a^4 + 7104785a^5 + 5406109a^6 + 2893077a^7 + 1109403a^8 + 306415a^9 + 60463a^{10} + 8315a^{11} + 757a^{12} + 41a^{13} + a^{14}$$

$$D(a) = a(2 + a)(5 + a)\left(3 + 5a + a^2\right)\left(8 + 7a + a^2\right)\left(11 + 7a + a^2\right)\left(19 + 9a + a^2\right)\left(4 + 22a + 25a^2 + 9a^3 + a^4\right)$$


$C_0(\mathbf{A1}, a)$
0.05
0.04
0.03
0.02
0.01
0
50
100
150
200

$$C_0(\mathrm{A2}, a) = \frac{N(a)}{D(a)}$$

$$N(a) = 440877645824 + 27367146861824a + 4621572918172 96a^2 + 405071465686182 2a^3 + 22613036510196994a^4 + 8895923557614619 7a^5 + 26193884126598534 3a^6 + 60076706913079021 9a^7 + 11039716243552962 07a^8 + 16596329645810608 09a^9 + 20738421775652612 37a^{10} + 21808663414903405 13a^{11} + 19490312218291103 35a^{12} + 14918227492197998 12a^{13} + 98400591827617322 2a^{14} + 56202490944891862 9a^{15} + 27899174499869787 5a^{16} + 1206888251509549 94a^{17} + 45576544679623618a^{18} + 15037924371814443a^{19} + 4335083301049651a^{20} + 1090850715403918a^{21} + 239144874716940a^{22} + 45538396282853a^{23} + 7499870089507a^{24} + 1062079338218a^{25} + 128325292364a^{26} + 13093056043a^{27} + 1112640113a^{28} + 77284163a^{29} + 4273197a^{30} + 180844a^{31} + 5500a^{32} + 107a^{33} + a^{34}$$

$$D(a) = a(2+a)(4+a)(5+a)\left(3+5a+a^2\right)\left(8+7a+a^2\right)\left(11+7a+a^2\right) \left(98+575a+821a^2+487a^3+139a^4+19a^5+a^6\right) \left(256+827a+955a^2+515a^3+141a^4+19a^5+a^6\right) \left(99848+937037a+3344594a^2+6391936a^3+7532510a^4+5894944a^5 + 3195866a^6+1227895a^7+337050a^8+65701a^9+8886a^{10}+793a^{11}+42a^{12}+a^{13}\right)$$

$$C_0(\mathrm{B1}, a) = \frac{N(a)}{D(a)}$$

$$N(a) = 11902464 + 656574240a + 7393099856a^2 + 40579763632a^3 + 137016635760a^4 + 317355282496a^5 + 536751276440a^6 + 689863664136a^7 + 692204761732a^8 + 552546748276a^9 + 355557565212a^{10} + 186115788200a^{11} + 79698949438a^{12} + 27996698426a^{13} + 8067218912a^{14} + 1901186464a^{15} + 364169759a^{16} + 56116363a^{17} + 6847455a^{18} + 646055a^{19} + 45423a^{20} + 2239a^{21} + 69a^{22} + a^{23}$$

$$D(a) = a(3+a)\left(6+6a+a^2\right)\left(14+8a+a^2\right)\left(10+18a+8a^2+a^3\right)\left(64+118a+66a^2+14a^3+a^4\right)\left(18+112a+142a^2+69a^3+14a^4+a^5\right)\left(246+830a+956a^2+515a^3+141a^4+19a^5+a^6\right)$$

$$C_0(\mathrm{B2}, a) = \frac{N(a)}{D(a)}$$

$$N(a) = 228403954279366656 00 + 176417052781141013504 0a +$$
$$4708753545228038038688 0a^2 + 7032545367757735395077 60a^3 +$$
$$7015710974339232710972144a^4 + 5121712133404624382584755 2a^5 +$$
$$2892275140179900020075250 24a^6 + 1311366445365745862261975728a^7 +$$
$$4903100211164892507453063984a^8 + 1542396813262984667793408723 2a^9 +$$
$$4146458710690228645530785425 6a^{10} + 9645474433160062036175617993 6a^{11} +$$
$$1961286148015429992814783677 60a^{12} + 3515416223954574086819769890 64a^{13} +$$
$$5593570600397637365329320853 68a^{14} + 7948195406874949491907273877 52a^{15} +$$
$$1013740859314758760921416650272a^{16} + 1165640809540507309899478188612a^{17} +$$
$$1212891021613116041168312811092a^{18} + 1145821608116283210822081738432a^{19} +$$
$$9855518645048951706457125704 72a^{20} + 7737006292071807048826517532 52a^{21} +$$
$$5555400851506579316015965997 68a^{22} + 3655061870403151471659157487 08a^{23} +$$
$$2206892865366154177473570148 40a^{24} + 1224440306792556286126391630 46a^{25} +$$
$$6249232338191125345455000729 6a^{26} + 2936414972831522968801892840 6a^{27} +$$
$$1271128656983382970468773581 8a^{28} + 5071496441284680229366229376a^{29} +$$

$$186537332836075919520468883 6a^{30} + 6325612591138890808273027 85a^{31} + 19773593245711464391067994 1a^{32} + 56959533345058437931721662a^{33} + 15111437884750072290446648a^{34} + 36895072290144780506572 40a^{35} + 82816303103157172723391 2a^{36} + 17068600276539474086910 8a^{37} + 32250816325291253575504a^{38} + 55761205861172450229 55a^{39} + 88024444249902493567 9a^{40} + 12653324099246972938 0a^{41} + 16511094603551091600a^{42} + 19485282835831201 30a^{43} + 20705500167904040 2a^{44} + 1970753105336015 4a^{45} + 166956785588472 0a^{46} + 124930903171831a^{47} + 8179497774003a^{48} + 463053194012a^{49} + 22324673532a^{50} + 898395016a^{51} + 29354004a^{52} + 747950a^{53} + 13940a^{54} + 169a^{55} + a^{56}$$

$$D(a) = a\big(44768 + 455938a + 1734518a^2 + 3527864a^3 + 4429788a^4 + 3698064a^5 + 2139272a^6 + 876477a^7 + 256170a^8 + 53055a^9 + 7604a^{10} + 717a^{11} + 40a^{12} + a^{13}\big)$$
$$\big(167880 + 1135190a + 3333882a^2 + 5657352a^3 + 6220108a^4 + 4701136a^5 + 2523260a^6 + 977337a^7 + 274050a^8 + 55099a^9 + 7740a^{10} + 721a^{11} + 40a^{12} + a^{13}\big)$$
$$\big(398720 + 4215458a + 17515520a^2 + 40405876a^3 + 59577850a^4 + 60447322a^5 + 44042916a^6 + 23631964a^7 + 9463582a^8 + 2839853a^9 + 635464a^{10} + 104419a^{11} + 12226a^{12} + 965a^{13} + 46a^{14} + a^{15}\big)$$
$$\big(457320 + 4434850a + 17873280a^2 + 40737960a^3 + 59770398a^4 + 60519398a^5 + 44060300a^6 + 23634568a^7 + 9463802a^8 + 2839861a^9 + 635464a^{10} + 104419a^{11} + 12226a^{12} + 965a^{13} + 46a^{14} + a^{15}\big)$$

$$C_0(\mathrm{B3}, a) = \frac{N(a)}{D(a)}$$

$$N(a) = 505382400 + 31085264896a + 496334043904a^2 + 4032822252736a^3 +$$
$$20523087501824a^4 + 724647900215 04a^5 + 1887006053648 00a^6 +$$
$$377329348010736a^7 + 59611951474294 4a^8 + 7597842151498 16a^9 +$$
$$7936507635830 88a^{10} + 687655134854992a^{11} + 498776098113408a^{12} +$$
$$304964841147766a^{13} + 157981759440416a^{14} + 69578309042138a^{15} +$$
$$26103588803056a^{16} + 8346647190203a^{17} + 2272484781652a^{18} +$$
$$525504601274a^{19} + 102771692118a^{20} + 16888769089a^{21} +$$
$$2310900456a^{22} + 259946974a^{23} + 23612684a^{24} + 1688201a^{25} +$$
$$91412a^{26} + 3522a^{27} + 86a^{28} + a^{29}$$

$$D(a) = a(2 + a)(3 + a)(5 + a)\left(36 + 88a + 56a^2 + 13a^3 + a^4\right)$$
$$\left(656 + 5640a + 15880a^2 + 21396a^3 + 15978a^4 +\right.$$
$$\left.7088a^5 + 1911a^6 + 307a^7 + 27a^8 + a^9\right)$$
$$\left(42800 + 402552a + 1483372a^2 + 2978576a^3 + 3734072a^4 + 3136628a^5 +\right.$$
$$\left.1836114a^6 + 764384a^7 + 227689a^8 + 48165a^9 + 7062a^{10} + 682a^{11} + 39a^{12} + a^{13}\right)$$

$$C_0(\mathrm{C1}, a) = \frac{N(a)}{D(a)}$$

$$N(a) = 285962070319707264 0 + 219353748972424670464a +$$
$$57478546265906491747 84a^2 + 83934659099726903357568a^3 +$$
$$81650323676958135713248 0a^4 + 57994149533918804721451 52a^5 +$$
$$31798306380943983167228736a^6 + 13971456331708453955295 9872a^7 +$$
$$50527134980867983456877966 4a^8 + 15345576210772810961763 40192a^9 +$$
$$39755843110522992613580576 00a^{10} + 88958691126759840089767 20336a^{11} +$$
$$17367958864888557662698636488a^{12} + 29834817385077250506106710048a^{13} +$$
$$45411029927441498643296659852a^{14} + 61608618168883559003731258424a^{15} +$$
$$74879240376109100817191535032a^{16} + 81885076784194859717999599048a^{17} +$$
$$80870815241049377301710367880a^{18} + 72363448966163901533355636428a^{19} +$$
$$58828916092690541813951352886a^{20} + 43555380199393908351697810443a^{21} +$$
$$29427818959285376710982899037a^{22} + 18175717412153506913791301396a^{23} +$$
$$10277139108717035633061825646a^{24} + 53262008488949180347893021 27a^{25} +$$
$$25324425260334890938879850 59a^{26} + 11054789748043534451126459 72a^{27} +$$
$$44326740491678405050140364 6a^{28} + 16330767760232112098132870 4a^{29} +$$
$$55284058144867627152787662a^{30} + 17194027516276283914174924a^{31} +$$
$$49110440119156554349384 50a^{32} + 12874208948908285927166 02a^{33} +$$

$$309486328284718689949200a^{34} + 681464855370383908418 74a^{35} + 137247866131642163143 58a^{36} + 252385774879995244844 5a^{37} + 422861320905237828945a^{38} + 643869567071472326 90a^{39} + 888268089435646782 2a^{40} + 110627550024798439 4a^{41} + 123845130699041042a^{42} + 12397752273003536a^{43} + 11029197792476 52a^{44} + 865325173318 95a^{45} + 5931621704951a^{46} + 351081953942a^{47} + 17673043212a^{48} + 741622600a^{49} + 25236840a^{50} + 668922a^{51} + 12954a^{52} + 163a^{53} + a^{54}$$

$$D(a) = a(2 + a)$$
$$(77190 + 383344a + 808656a^2 + 968855a^3 + 738553a^4 + 378242a^5 + 133356a^6 + 32475a^7 + 5368a^8 + 575a^9 + 36a^{10} + a^{11})$$
$$(12796 + 127138a + 459112a^2 + 868524a^3 + 996257a^4 + 746079a^5 + 379464a^6 + 133464a^7 + 32479a^8 + 5368a^9 + 575a^{10} + 36a^{11} + a^{12})$$
$$(122520 + 1325504a + 6041028a^2 + 15619080a^3 + 25903278a^4 + 29440464a^5 + 23854840a^6 + 14113875a^7 + 6178453a^8 + 2009898a^9 + 483788a^{10} + 84911a^{11} + 10552a^{12} + 879a^{13} + 44a^{14} + a^{15})$$
$$(708904 + 7249920a + 29235644a^2 + 65418908a^3 + 93378986a^4 + 91499228a^5 + 64241260a^6 + 33151505a^7 + 12748827a^8 + 3669812a^9 + 787140a^{10} + 123927a^{11} + 13900a^{12} + 1051a^{13} + 48a^{14} + a^{15})$$

$$C_0(\mathrm{C2}, a) = \frac{\max\{N_1(a), N_5(a)\}}{D(a)}$$

$$N_1(a) = 183380756790400056448 + 143272254577259114690 24a + 389111466243940898232480a^2 + 593235273080318506891 9812a^3 + 605684481074631156941880 90a^4 + 453558924841917655390413700a^5 + 263274079653912743074894 8968a^6 + 122941203211843254467890501 52a^7 + 474316892920607456768890505 04a^8 + 154245783546065507965546716804a^9 + 429429789101003375384118323540a^{10} + 103634383981820696031259116 4030a^{11} + 219002474728027008986881317 6902a^{12} + 408670814679972812050057298 3296a^{13} + 678169308339343127620655020 0228a^{14} + 100678916957719560825561861 63176a^{15} + 134398772893129194745763169815 42a^{16} + 162039219687285030257217707 98526a^{17} + 177119761646791782305844645557 10a^{18} + 176105022377708050787129386 31190a^{19} + 159728742191217275529556192647 12a^{20} + 132491314043012530820383431 08661a^{21} + 100722972743291663811023252062 71a^{22} + 703110378483044589002538974 6370a^{23} + 451414440764875595852919339895 2a^{24} + 266921675884172990915807444404 6a^{25} + 145530088911821847630774154869 8a^{26} + 732314323613305793300330432296a^{27} + 340368544817210325430548099010a^{28} + 146204148375485407475430878552a^{29} +$$

$$580633357780440316405851163 16a^{30} + 213242473820331553073019911 42a^{31} +$$
$$724263308605675731421732357 8a^{32} + 22746541006118606646502389 89a^{33} +$$
$$660384819542341136238687939a^{34} + 17714424977434134366234565 0a^{35} +$$
$$438737903168695078591157 64a^{36} + 10023974103170490524717242a^{37} +$$
$$211028566088873769155422 0a^{38} + 40879987456277940379367 8a^{39} +$$
$$72749496281487886338920a^{40} + 11869853246826088399567a^{41} +$$
$$17715326511466025534 23a^{42} + 24118803526356363353 8a^{43} +$$
$$29858704339512042430a^{44} + 334851854305001542 4a^{45} +$$
$$338656730882189476a^{46} + 30724385415081638a^{47} +$$
$$2484589257129642a^{48} + 177712315368245a^{49} +$$
$$11136414326333a^{50} + 6041883953 32a^{51} +$$
$$27949875660a^{52} + 1080502640a^{53} +$$
$$33953224a^{54} + 832942a^{55} + 14962a^{56} + 175a^{57} + a^{58}$$

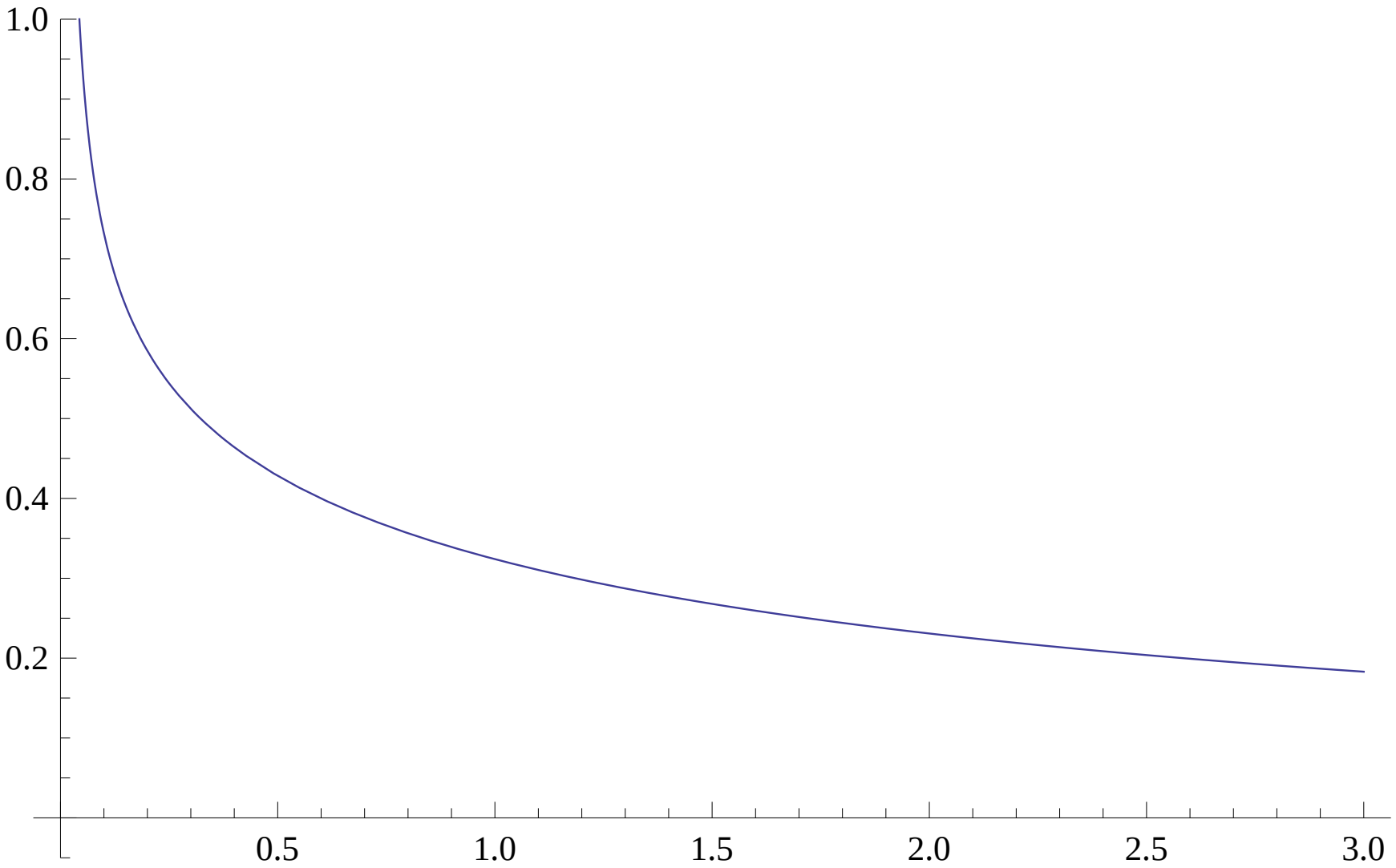
$$N_5(a) = 183380756790400056448 + 143243723013356495367 36a + 389223899392115934466848a^2 + 59363717997600668917823 88a^3 + 60623418695858945200753050a^4 + 45401720031251763273937 2472a^5 + 26354399058293142536987803 92a^6 + 12306219612160525421256126720a^7 + 47474883042624259009457386048a^8 + 15437239442541227665972559756 8a^9 + 42974129208291171216264519526 6a^{10} + 10369978567540359416837986936 98a^{11} + 21912117472354040829204074960 28a^{12} + 40885896860787047036362895379 36a^{13} + 67843197553063612090999700164 30a^{14} + 10071143464131398547484780120524a^{15} + 13443467720670510149927280730072a^{16} + 16207474835708906462530252225026a^{17} + 17715139763421202388227532538364a^{18} + 17613045821223701184066740312660a^{19} + 15974726185870821346635857044676a^{20} + 13250355466332378033388714614675a^{21} + 10073033212783634741189276804355a^{22} + 70315069427850299272113272873 02a^{23} + 45143459145143809382942493879 94a^{24} + 26693087484224142966426921627 74a^{25} + 14553392731595156961164109042 82a^{26} + 73232897018519396650946333017 0a^{27} + 34037365678435706712337359322 6a^{28} + 14620578024145392669394811250 2a^{29} +$$

$$580638120830420124930269986 12a^{30} + 213243744183913700284992277 40a^{31} +$$
$$724266401910480211934191131 8a^{32} + 227466096877652472611317180 9a^{33} +$$
$$660386207903676706903416623a^{34} + 177144504786294785696669172a^{35} +$$
$$438738327765397385430325 92a^{36} + 100239804930523392769782 76a^{37} +$$
$$211028652703913138382568 8a^{38} + 408799979868960474162454a^{39} +$$
$$727495077069728104785 56a^{40} + 118698543463091563009 85a^{41} +$$
$$177153274428611077658 3a^{42} + 241188042144900988082a^{43} +$$
$$298587047777626221 62a^{44} + 334851856674967691 6a^{45} +$$
$$338656731948940164a^{46} + 30724385453961484a^{47} +$$
$$24845892582318 44a^{48} + 177712315391043a^{49} +$$
$$11136414326639a^{50} + 604188395334a^{51} +$$
$$27949875660a^{52} + 1080502640a^{53} +$$
$$33953224a^{54} + 832942a^{55} + 14962a^{56} + 175a^{57} + a^{58}$$

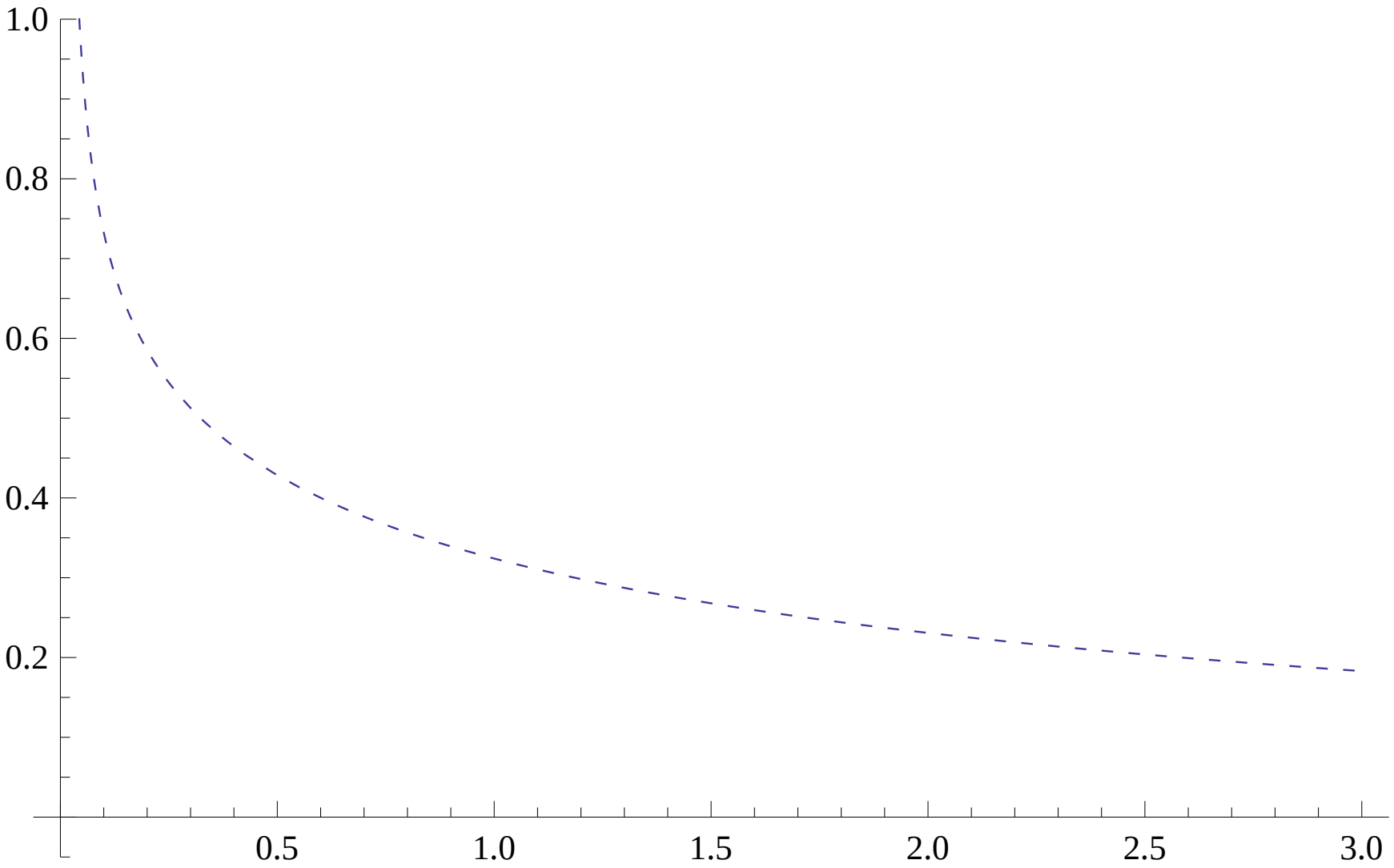
$$D(a) = 1100284540742400338688 0a+$$
$$41722222700624794600345 6a^2 + 75164695701629075812563 84a^3+$$
$$85943764591090932637051416a^4 + 70292902057979341566621 0172a^5+$$
$$43939611206224615228243314 72a^6 + 21898757804923309272760617704a^7+$$
$$89618025016863045004609550576a^8 + 30776488998908034713212311 7632a^9+$$
$$90185548461323379982322877 4592a^{10} + 22849896629424265798726484 42760a^{11}+$$
$$50594795995340021830176006 55616a^{12} + 98771575784530488862768119 13612a^{13}+$$
$$17126416863275728951668443346944a^{14} + 26541179095209836066597895872488a^{15}+$$
$$36957936816471850827618235208420a^{16} + 46453637085054034897782248282092a^{17}+$$
$$52914882674534260610919545805060a^{18} + 54812165375736560428898365344508a^{19}+$$
$$51786723928463161169320551724 68a^{20} + 44744158173836870204017888643408a^{21}+$$
$$35433857907896107385448939124838a^{22} + 25770513606441900150105494930661a^{23}+$$
$$17242204124247732908853250817652a^{24} + 10628336484609505804469450079712a^{25}+$$
$$60434147638239782801723106728 80a^{26} + 31731968786910759827669754514 44a^{27}+$$
$$15398511922264619670508752746 88a^{28} + 69106379163241252807736968179 4a^{29}+$$

$$28696550288269841751952903470 4a^{30} + 11029451144169294864706694669 6a^{31} +$$
$$39242637646954919980609253008a^{32} + 12925188584771425569234935390a^{33} +$$
$$39401268085957634242587247 38a^{34} + 11112764751312493230715851 09a^{35} +$$
$$28982592694695417637888356 8a^{36} + 69845146785181725384925326a^{37} +$$
$$15538533646096332188604808a^{38} + 31875144643663498207965 06a^{39} +$$
$$60207305772908572017475 6a^{40} + 10453666743966093529937 8a^{41} +$$
$$16651137654805893093310a^{42} + 24274773346823150957 71a^{43} +$$
$$32300221256439074230 4a^{44} + 39101201594191028904a^{45} +$$
$$42899978177810451 84a^{46} + 4246741660602510 52a^{47} +$$
$$3772845009439355 2a^{48} + 298895716101331 2a^{49} +$$
$$2095289127652 86a^{50} + 12873835314783a^{51} +$$
$$685076473320a^{52} + 31096549020a^{53} +$$
$$1179992500a^{54} + 36408706a^{55} + 877312a^{56} + 15484a^{57} + 178a^{58} + a^{59}$$

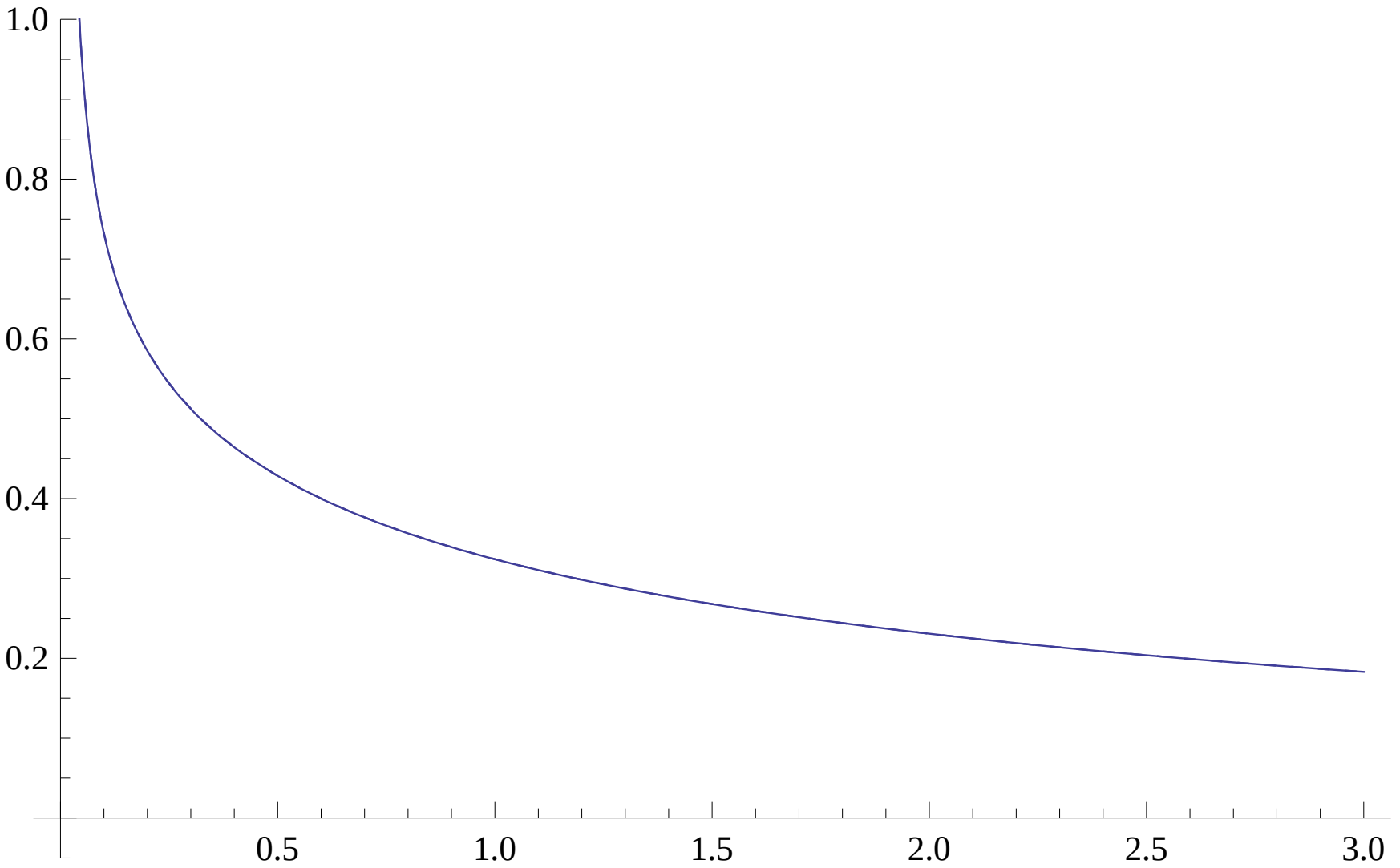
$$C_0(\mathrm{C2}, a) = \frac{N_1(a)}{D(a)}$$



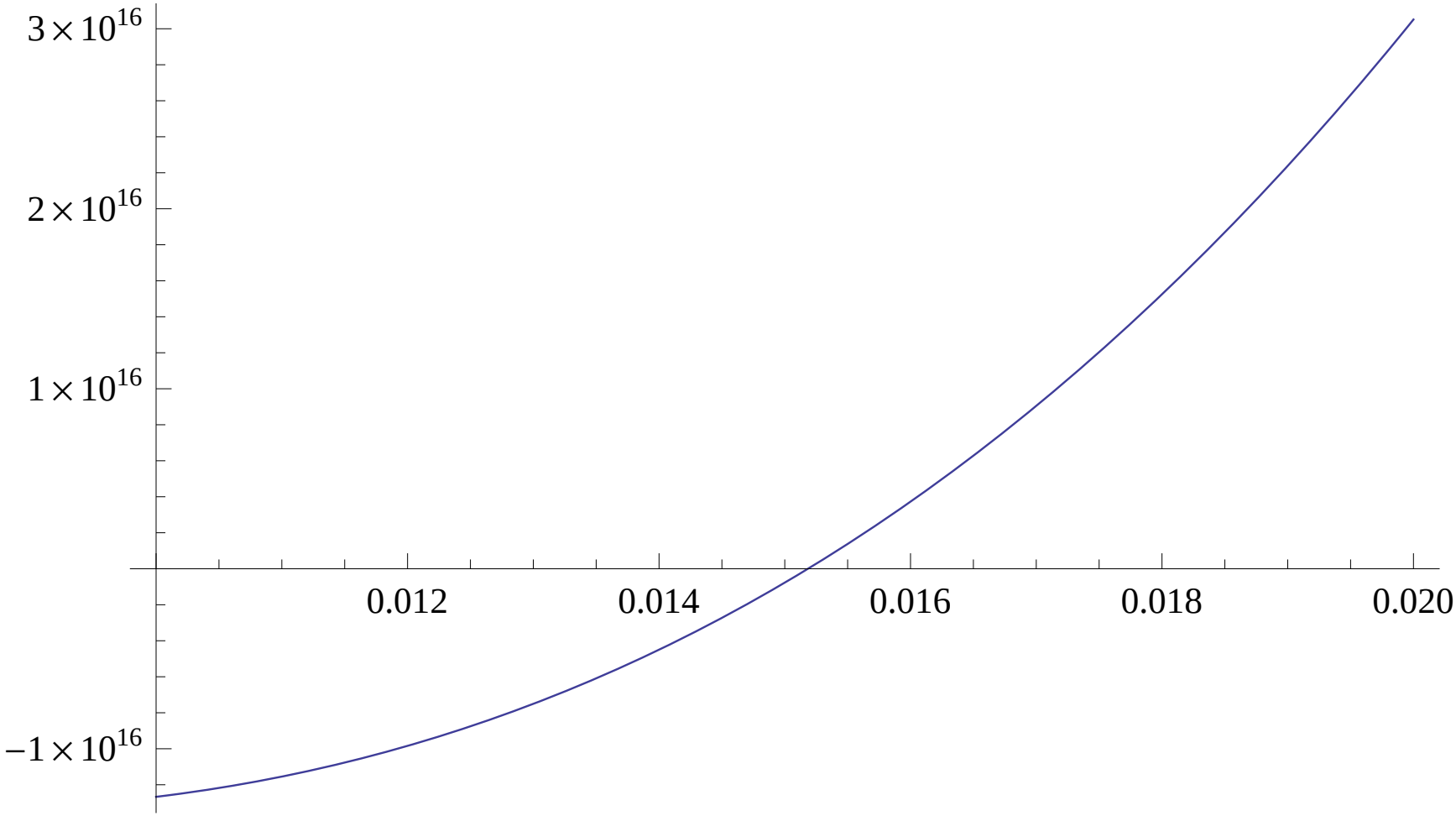
$$C_0(\mathrm{C2}, a) = \frac{N_5(a)}{D(a)}$$



$$C_0(\mathrm{C2}, a) = \max\left\{\frac{N_1(a)}{D(a)}, \frac{N_5(a)}{D(a)}\right\}$$



$$N_5(a) - N_1(a)$$



$$a_0 = 0.0151$$

$$C_0(\mathrm{C3}, a) = \frac{N(a)}{D(a)}$$

$$N(a) = 17919527230638720 0 + 1350739915257042720 0a +$$
$$34262297280444938644 8a^2 + 48141224118704353559 96a^3 +$$
$$44861420104232528149894a^4 + 30404607810989791948648 2a^5 +$$
$$15849065447934836405278 01a^6 + 65970291691804738047057 37a^7 +$$
$$22523590790210718433323308a^8 + 64361343369372864904428854a^9 +$$
$$15635395809137153734193468 3a^{10} + 32696892564973506266482786 9a^{11} +$$
$$59459629393819832758716580 6a^{12} + 94818424695917155067934100 0a^{13} +$$
$$13352347508798150649553741 31a^{14} + 16702485253641746849912842 63a^{15} +$$
$$18652686121493821871743219 56a^{16} + 18676587665160991897476367 00a^{17} +$$
$$16828297878713597895900139 62a^{18} + 13687707643257245539779698 94a^{19} +$$
$$10077007520707119704324538 38a^{20} + 67302329591194725502745623 2a^{21} +$$
$$40856109819173154232525423 2a^{22} + 22578804830566377398431312 6a^{23} +$$
$$11374163726559502174070110 6a^{24} + 52281415991370048833585734a^{25} +$$
$$21943316801536574734628687a^{26} + 84137297227178164912372 65a^{27} +$$
$$29478037780263443749618 82a^{28} + 94365297041389696952685 2a^{29} +$$

$$27592742501438768354506 6a^{30} + 736532172562889577542 82a^{31} +$$
$$17931706027226055468360a^{32} + 39771007062493854623 46a^{33} +$$
$$80234054295664165871 9a^{34} + 14694768755091248585 7a^{35} +$$
$$24375488265749194878a^{36} + 365162125759312097 0a^{37} +$$
$$49233137978059683 2a^{38} + 5949192406511334 8a^{39} +$$
$$6410632326020116a^{40} + 612255089129920a^{41} +$$
$$5144039019802 3a^{42} + 3766964373975a^{43} +$$
$$237642565878a^{44} + 12722551532a^{45} +$$
$$566610832a^{46} + 20422448a^{47} +$$
$$572254a^{48} + 11694a^{49} + 155a^{50} + a^{51}$$

$$D(a) = a(2+a)(4+a)$$
$$(557660340 + 8329233813a + 55127691398a^2 +$$
$$218090879535a^3 + 583814567550a^4 + 1131872007137a^5 +$$
$$1658340411710a^6 + 1889549591178a^7 + 1708486233852a^8 +$$
$$1243674906498a^9 + 736388512220a^{10} + 357152561775a^{11} +$$
$$142491170544a^{12} + 46843843732a^{13} + 12679508042a^{14} +$$
$$2815931075a^{15} + 509718844a^{16} + 74409012a^{17} +$$
$$8620944a^{18} + 773907a^{19} + 51872a^{20} + 2442a^{21} + 72a^{22} + a^{23})$$
$$(2410005600 + 46931818200a + 403273714784a^2 + 2054033992636a^3 +$$
$$7034425363933a^4 + 17393056574982a^5 + 32499548833503a^6 +$$
$$47357574728470a^7 + 55046736592289a^8 + 51897017413050a^9 +$$
$$40180314421430a^{10} + 25783308356500a^{11} + 13803794421854a^{12} +$$
$$6193483471180a^{13} + 2334821012579a^{14} + 740058368400a^{15} +$$
$$197006379064a^{16} + 43907214310a^{17} + 8148340939a^{18} + 1248770268a^{19} +$$
$$156156668a^{20} + 15661144a^{21} + 1228523a^{22} + 72568a^{23} + 3034a^{24} + 80a^{25} + a^{26})$$

$$C_0(\mathrm{C4}, a) = \frac{N(a)}{D(a)}$$

$$N(a) = (4+a)\big(143149761103951641 6 + 1101194516751575149 44a +$$
$$2902816365100755175560a^2 + 4269468459373465625088 4a^3 +$$
$$4186692441617411721257 84a^4 + 2999685406120426737007954a^5 +$$
$$1660100311204720716517328 0a^6 + 7366223793133599982887378 8a^7 +$$
$$2691622766503039617177881 40a^8 + 8263331828837095643054916 33a^9 +$$
$$2164894348023478134084285627a^{10} + 4900674328825817073411901982a^{11} +$$
$$9682863323985793944491918444a^{12} + 1683877689650837035586326313 1a^{13} +$$
$$2595461698375715416799504158 3a^{14} + 3566849155249256542450980540 6a^{15} +$$
$$4392471256273231444390655738 6a^{16} + 4868121186159107880818347990 5a^{17} +$$
$$4873639925272762895363319685 1a^{18} + 4421534242219624641403770970 2a^{19} +$$
$$3645149156011608596396234458 6a^{20} + 2737204928672553010485839466 3a^{21} +$$
$$1875974342953036158014501504 5a^{22} + 1175487802912356847152848476 8a^{23} +$$
$$6743779543386975325271042438a^{24} + 3546440357772184748633355767a^{25} +$$
$$1711161396883371443200743565a^{26} + 7580597723227364277634281 94a^{27} +$$
$$3084885036404690364839472 96a^{28} + 1153485642276070554533984 89a^{29} +$$

$$396318074344508509421530 99a^{30} + 125100764385307837334334 46a^{31} + 362650977576993765214873 2a^{32} + 964844006526455835776 44a^{33} + 235387386238086929329088a^{34} + 525979782805946012922 06a^{35} + 107495201207252975252 10a^{36} + 200574549751180965414 7a^{37} + 340958723274768320507a^{38} + 526691277611959476 94a^{39} + 737079768366622234 6a^{40} + 931108552796557666a^{41} + 105714302212377268a^{42} + 10731583258167712a^{43} + 968002143332782a^{44} + 76995667644665a^{45} + 5350023274283a^{46} + 320940584638a^{47} + 16371863932a^{48} + 696106120a^{49} + 23997540a^{50} + 644286a^{51} + 12636a^{52} + 161a^{53} + a^{54})$$

$$D(a) = a(2+a)\left(3+5a+a^2\right)\left(11+7a+a^2\right)$$
$$\left(129023280+1762939170a+10373152354a^2+35856778041a^3+82868029210a^4+\right.$$
$$137412738354a^5+170803393706a^6+163832764158a^7+123702404356a^8+$$
$$74535044543a^9+36163118046a^{10}+14202908986a^{11}+4523997070a^{12}+$$
$$1167075464a^{13}+242634794a^{14}+40278230a^{15}+5259974a^{16}+$$
$$\left.528041a^{17}+39298a^{18}+2041a^{19}+66a^{20}+a^{21}\right)$$
$$\left(40345012608+825626945888a+7581731395544a^2+41967626810112a^3+\right.$$
$$1586252724716 04a^4+439076583569846a^5+93088629626146 0a^6+$$
$$155919206136430 3a^7+21100579374960 18a^8+23461210768960 25a^9+$$
$$21706259857911 00a^{10}+16873764986740 81a^{11}+11102772880223 74a^{12}+$$
$$6217560711733 31a^{13}+2974785754784 22a^{14}+1218976958786 91a^{15}+$$
$$4282693632447 4a^{16}+1289817390902 9a^{17}+3324656188818a^{18}+$$
$$731213931104a^{19}+136568040684a^{20}+21512223851a^{21}+2830903138a^{22}+$$
$$\left.307177372a^{23}+26989232a^{24}+1871027a^{25}+98456a^{26}+3694a^{27}+88a^{28}+a^{29}\right)$$

$$C_0(\mathrm{C5}, a) = \frac{N(a)}{D(a)}$$

$$N(a) = 2287563586172359065 6 + 1771610487172824563200a +$$
$$4729553225238703675795 2a^2 + 7061342640045657675401 92a^3 +$$
$$7041144711763026185479664a^4 + 5137752963233760483160044 8a^5 +$$
$$2899979813101426831232210 48a^6 + 1314304768485423455922439456a^7 +$$
$$4912261136265904407564189656a^8 + 1544782560825909003851162414 4a^9 +$$
$$4151734847231737847085561771 2a^{10} + 9655512567589751936227207283 2a^{11} +$$
$$1962946390613916623382244394 12a^{12} + 3517823805225700407945904413 32a^{13} +$$
$$5596653453863310739953345807 42a^{14} + 7951701755861173627271852883 20a^{15} +$$
$$1014096858256264569675182776448a^{16} + 1165964820413003396200687086018a^{17} +$$
$$1213156318938009241669984613834a^{18} + 1146017616204282046208611186236a^{19} +$$
$$9856828672428137617704548682 44a^{20} + 7737800009399242996175897820 44a^{21} +$$
$$5555837547808708105217892777 56a^{22} + 3655280354513521371746920711 97a^{23} +$$
$$2206992370489200880399090718 07a^{24} + 1224481590836509007430903017 48a^{25} +$$
$$6249388447288934515136428550 8a^{26} + 2936468781892092457054468893 5a^{27} +$$
$$1271145561758789465266046110 1a^{28} + 5071544825792904104585323904a^{29} +$$

$$186538593580733397513120473 4a^{30} + 6325642466201226694842547 00a^{31} +$$
$$197736575340391245168070066a^{32} + 56959658747205651441339900a^{33} +$$
$$151114600075711582106138 04a^{34} + 3689510748898460409794054a^{35} +$$
$$828163534412499730423766a^{36} + 17068606720643902997603 6a^{37} +$$
$$322508236732066963275 62a^{38} + 55761213278885953840 85a^{39} +$$
$$880244508302974268979a^{40} + 126533246075269947026a^{41} +$$
$$16511094941436703038a^{42} + 19485283026262589 38a^{43} +$$
$$207055002571043474a^{44} + 19707531087144608a^{45} +$$
$$16695678568786 30a^{46} + 124930903193137a^{47} +$$
$$8179497774299a^{48} + 463053194014a^{49} +$$
$$22324673532a^{50} + 898395016a^{51} +$$
$$29354004a^{52} + 747950a^{53} + 13940a^{54} + 169a^{55} + a^{56}$$

$$D(a) = a(2+a)$$
$$\left(33922169760 + 568756091408a + 4273869055244a^2 + \right.$$
$$19453794780368a^3 + 60804681647648a^4 + 139829058004980a^5 +$$
$$247089900601620a^6 + 345580569056844a^7 + 390705471315120a^8 +$$
$$362657726468648a^9 + 279587461514156a^{10} + 180573673073441a^{11} +$$
$$98318887715262a^{12} + 45326492595349a^{13} + 17739674184940a^{14} +$$
$$5900285743270a^{15} + 1666891356182a^{16} + 399120397310a^{17} + 80670983362a^{18} +$$
$$13678962279a^{19} + 1928530822a^{20} + 223239502a^{21} + 20844212a^{22} +$$
$$\left.1530331a^{23} + 85016a^{24} + 3358a^{25} + 84a^{26} + a^{27}\right)$$
$$\left(20230695168 + 403204324480a + 3589597495544a^2 + 1917803541974 8a^3 + \right.$$
$$69662442299856a^4 + 184530285123584a^5 + 37283873528785 2a^6 +$$
$$5927033887709 24a^7 + 7581741538284 44a^8 + 7935540143582 24a^9 +$$
$$6882566078735 24a^{10} + 4994094586068 30a^{11} + 3053678614333 85a^{12} +$$
$$1581703856573 84a^{13} + 6964712155356 9a^{14} + 2612361271256 4a^{15} +$$
$$8351335336618a^{16} + 2273368119446a^{17} + 525637688246a^{18} +$$
$$102787513060a^{19} + 16890220371a^{20} + 2310999566a^{21} + 259951718a^{22} +$$
$$\left.23612826a^{23} + 1688203a^{24} + 91412a^{25} + 3522a^{26} + 86a^{27} + a^{28}\right)$$

$$C_0(\mathrm{C6}, a) = \frac{N(a)}{D(a)}$$

$$N(a) = 4582563713927844739 2 + 3571720928625000378544a +$$
$$964657891548867330597 56a^2 + 1460556869037499772288898a^3 +$$
$$14794657855859119385302264a^4 + 109824488413752952126647278a^5 +$$
$$631471877513175539827717944a^6 + 291888014111686198767750881 4a^7 +$$
$$1113944494165603031474462554 5a^8 + 3580907976836781602876774740 9a^9 +$$
$$9848438668333762197068999684 8a^{10} + 2346301407297906658313733377 52a^{11} +$$
$$4891473801304384927593967668 18a^{12} + 8998624071008588571198511236 08a^{13} +$$
$$1471115691766237828549717079166a^{14} + 2150007656634107012966583964428a^{15} +$$
$$2823365133138824519921011957686a^{16} + 3346027141913037317235406822614a^{17} +$$
$$3592277705152966160810694994774a^{18} + 3505191592548820162783113540596a^{19} +$$
$$3117383316076815204537253660377a^{20} + 2533242824771056675548373138627a^{21} +$$
$$1884947478908222967223912799458a^{22} + 1286639808291688970188083744260a^{23} +$$
$$806926501539897904858160751680a^{24} + 4655955520313893650419276478 04a^{25} +$$
$$2474359269187708623622471551 44a^{26} + 1212232597681764023552365676 10a^{27} +$$
$$5478759896022407198846217704 6a^{28} + 2285454524626497097211948895 8a^{29} +$$

$$880231669242553930354834256 2a^{30} + 313051442808264996390893471 0a^{31} +$$
$$102803739713474545275596059 2a^{32} + 311655892809434391229490018a^{33} +$$
$$87182578758927492955100790a^{34} + 22490481504962881777677606a^{35} +$$
$$53458378176815692021905 44a^{36} + 116953566882163065520813 2a^{37} +$$
$$23518771455256561694893 2a^{38} + 43403066328965374526524a^{39} +$$
$$73366103444935556751 93a^{40} + 11333110004534717011 25a^{41} +$$
$$15955543595814194199 4a^{42} + 204081515466891868 80a^{43} +$$
$$23626383077683453 56a^{44} + 24647006474533268 6a^{45} +$$
$$2304666680090186 8a^{46} + 191942966326490 4a^{47} +$$
$$14128989874173 5a^{48} + 9105604162327a^{49} +$$
$$507702541078a^{50} + 24121463564a^{51} +$$
$$957103024a^{52} + 30849904a^{53} + 775830a^{54} + 14278a^{55} + 171a^{56} + a^{57}$$

$$D(a) = a(2+a)$$
$$\left(34967090610 + 579606010332a + 4325898016546a^2 + \right.$$
$$19607032184489a^3 + 61115828159474a^4 + 14029264519241 0a^5 +$$
$$2476166240134 82a^6 + 3460487274026 60a^7 + 3910367042818 24a^8 +$$
$$362846510882591a^9 + 279674805272550a^{10} + 180606616413596a^{11} +$$
$$98329027861608a^{12} + 45329034539346a^{13} + 17740190280910a^{14} +$$
$$5900369787644a^{15} + 1666902166646a^{16} + 399121470172a^{17} +$$
$$80671062558a^{18} + 13678966371a^{19} + 1928530954a^{20} +$$
$$223239504a^{21} + 20844212a^{22} + 1530331a^{23} +$$
$$\left.85016a^{24} + 3358a^{25} + 84a^{26} + a^{27}\right)$$
$$\left(39316085216 + 809869928856a + 7475320160542a^2 + \right.$$
$$41538241498812a^3 + 1574519525960 22a^4 + 436751531963945a^5 +$$
$$92739869640319 0a^6 + 1555115669940542a^7 + 2106268988124 94a^8 +$$
$$23432794948465 96a^9 + 2168888777345920a^{10} + 1686504751944827a^{11} +$$
$$11099167551676 30a^{12} + 6216329962729 80a^{13} + 2974439321477 96a^{14} +$$
$$12188968508975 0a^{15} + 4282542502379 8a^{16} + 1289794379588 0a^{17} +$$
$$3324628366186a^{18} + 731211323744a^{19} + 136567858222a^{20} +$$
$$21512214883a^{21} + 2830902862a^{22} + 307177368a^{23} +$$
$$\left.26989232a^{24} + 1871027a^{25} + 98456a^{26} + 3694a^{27} + 88a^{28} + a^{29}\right)$$

$$C_0(\mathrm{C7}, a) = \frac{N(a)}{D(a)}$$

$$N(a) = 4590467651063162880 0 + 3580108146837440849664a +$$
$$9664053905279460975004 8a^2 + 1462432520052185252297760a^3 +$$
$$1480775290474536275214744 0a^4 + 1098906775747607995177228 80a^5 +$$
$$6317294234440074230766841 04a^6 + 2919682802221673916447416284a^7 +$$
$$1114150464790798955880573655 2a^8 + 3581352047646074080064366793 4a^9 +$$
$$9849255518944214611727950444 4a^{10} + 2346431150349442639057772108 98a^{11} +$$
$$4891653451292140910920331950 23a^{12} + 8998842611272588469430867340 27a^{13} +$$
$$1471139196048985712658940831296a^{14} + 2150030123472252554838390023410a^{15} +$$
$$2823384301912736535711849549935a^{16} + 3346041792643608900247024674701a^{17} +$$
$$3592287765639787079263904775970a^{18} + 3505197814406056542469820192040a^{19} +$$
$$3117386788248606832798301675585a^{20} + 2533244575883667577762952087025a^{21} +$$
$$1884948277891746727964377046968a^{22} + 1286640138348845623103474625150a^{23} +$$
$$8069266250288474519447181801 26a^{24} + 4655955938779698612116782835 62a^{25} +$$
$$2474359397578445220051092610 46a^{26} + 1212232633321641396210851433 38a^{27} +$$
$$5478759985431744859662839527 2a^{28} + 2285454544866152137560763411 6a^{29} +$$

$$880231673368467996441541791 0a^{30} + 313051443563746825065627143 0a^{31} + 10280373983733556498881026 77a^{32} + 311655892990548868296272799a^{33} + 87182578782434406742720870a^{34} + 22490481507655051455128530a^{35} + 53458378179516613813934 90a^{36} + 11695356688451529497664 06a^{37} + 23518771455432358070484 0a^{38} + 43403066329076456530338a^{39} + 73366103444993738794 09a^{40} + 11333110004537174873 69a^{41} + 15955543595814998747 2a^{42} + 204081515466893783 32a^{43} + 23626383077683483 02a^{44} + 24647006474533270 8a^{45} + 2304666680090186 8a^{46} + 191942966326490 4a^{47} + 14128989874173 5a^{48} + 9105604162327a^{49} + 507702541078a^{50} + 24121463564a^{51} + 957103024a^{52} + 30849904a^{53} + 775830a^{54} + 14278a^{55} + 171a^{56} + a^{57}$$

$$D(a) = a(2+a)$$
$$\left(36261801000 + 591286905480a + 4374287654760a^2 + \right.$$
$$19729719074656a^3 + 613298463338 28a^4 + 140566290646385a^5 +$$
$$24788325648691 8a^6 + 34625183081828 3a^7 + 3911597453236 58a^8 +$$
$$3629064648881 61a^9 + 2796984628094 38a^{10} + 1806141983987 78a^{11} +$$
$$9833100010217 6a^{12} + 453294490564 70a^{13} + 17740260045104a^{14} +$$
$$5900379052075a^{15} + 1666903115252a^{16} + 399121542392a^{17} +$$
$$80671066406a^{18} + 13678966499a^{19} + 1928530956a^{20} +$$
$$223239504a^{21} + 20844212a^{22} + 1530331a^{23} + 85016a^{24} + 3358a^{25} + 84a^{26} + a^{27}\left.\right)$$
$$\left(37977713664 + 791854091040a + 7371991048648a^2 + 411896566726 80a^3 + \right.$$
$$15666082613339 2a^4 + 435452008477136a^5 + 92578230587373 6a^6 +$$
$$15535469478270 81a^7 + 21050563260188 58a^8 + 23425220435884 59a^9 +$$
$$21685027457131 50a^{10} + 16863432409226 01a^{11} + 11098611138255 38a^{12} +$$
$$6216172111764 54a^{13} + 29744025518439 2a^{14} + 12188898621165 0a^{15} +$$
$$4282531775641 2a^{16} + 12897930710999a^{17} + 3324627127756a^{18} +$$
$$7312112361 16a^{19} + 136567853862a^{20} + 21512214747a^{21} + 2830902860a^{22} +$$
$$307177368a^{23} + 26989232a^{24} + 1871027a^{25} + 98456a^{26} + 3694a^{27} + 88a^{28} + a^{29}\left.\right)$$

$$C_0(\mathrm{C8}, a) = \frac{N(a)}{D(a)}$$

$$N(a) = 2289241259396130816 0 + 1775544737442544581120a +$$
$$47380027568607716488576a^2 + 70702649655857884427792 0a^3 +$$
$$70471883683640055716572 32a^4 + 5140698820175032051415119 0a^5 +$$
$$29010818017570330947520320 2a^6 + 1314634425868012245088464969a^7 +$$
$$49130725370386868189526023 23a^8 + 1544950335495904965680224164 0a^9 +$$
$$41520309122537301632180684386a^{10} + 96559639681711433620768227965a^{11} +$$
$$19630064399360765388564691517a^{12} + 35178940587192646220336370308 8a^{13} +$$
$$55967262078935754769362507604 6a^{14} + 79517688052134400024120202903 3a^{15} +$$
$$10141023815407730987563015488 63a^{16} + 11659689021906951248633648772 36a^{17} +$$
$$12131590332043648069390617221 96a^{18} + 11460192442461930484528300062 75a^{19} +$$
$$98568374978016897094189760444 3a^{20} + 77378043395837316671713014401 0a^{21} +$$
$$55558394729356565264419309219 6a^{22} + 36552811305801489853163092936 7a^{23} +$$
$$22069926542626662079327638139 3a^{24} + 12244816849530993581724156083 8a^{25} +$$
$$62493887303052639779169774322a^{26} + 29364688589953536911552371871a^{27} +$$
$$12711455807670132537894571203a^{28} + 50715448681300934170647214 66a^{29} +$$

$$186538594430914048632299233 2a^{30} + 6325642481553477719598743 54a^{31} +$$
$$19773657558887454496958884 8a^{32} + 56959658783111094524504218a^{33} +$$
$$15111460012180618416440646a^{34} + 36895107494210732462422 30a^{35} +$$
$$82816353446444695818150 6a^{36} + 17068606721092469381672 2a^{37} +$$
$$32250823673539319781966a^{38} + 55761213279094622907 85a^{39} +$$
$$88024450830406001330 1a^{40} + 12653324607531553514 4a^{41} +$$
$$16511094941438186964a^{42} + 19485283026262940 68a^{43} +$$
$$20705500257104401 2a^{44} + 1970753108714461 2a^{45} +$$
$$16695678568786 30a^{46} + 124930903193137a^{47} +$$
$$8179497774299a^{48} + 463053194014a^{49} +$$
$$22324673532a^{50} + 898395016a^{51} +$$
$$29354004a^{52} + 747950a^{53} + 13940a^{54} + 169a^{55} + a^{56}$$

$$D(a) = a(2+a)\big(37246511880 + 600024377592a + 4409642491698a^2 +$$
$$19816704103799a^3 + 61476159405824a^4 + 14074552168651 1a^5 +$$
$$2480494455662 62a^6 + 346371423937785a^7 + 39122763945586 4a^8 +$$
$$3629371855651 67a^9 + 279709601981724a^{10} + 18061743877584 3a^{11} +$$
$$9833175400015 6a^{12} + 45329588241609a^{13} + 1774028015570 6a^{14} +$$
$$5900381276062a^{15} + 1666903296900a^{16} + 399121552712a^{17} +$$
$$80671066770a^{18} + 13678966505a^{19} + 1928530956a^{20} +$$
$$223239504a^{21} + 20844212a^{22} + 1530331a^{23} +$$
$$85016a^{24} + 3358a^{25} + 84a^{26} + a^{27}\big)$$
$$\big(18438568960 + 379452471680a + 3455874909952a^2 +$$
$$187352827983 56a^3 + 686754501031 07a^4 + 182935985726254a^5 +$$
$$3708861641134 95a^6 + 59083513224318 8a^7 + 7567486223001 93a^8 +$$
$$7926742047372 82a^9 + 6878131822477 15a^{10} + 4992258747283 86a^{11} +$$
$$3053052528090 95a^{12} + 1581528000162 30a^{13} + 6964306598196 9a^{14} +$$
$$2612284973581 4a^{15} + 8351219471894a^{16} + 2273354142916a^{17} +$$
$$5256363809 32a^{18} + 102787421704a^{19} + 16890215885a^{20} +$$
$$2310999428a^{21} + 259951716a^{22} + 23612826a^{23} +$$
$$1688203a^{24} + 91412a^{25} + 3522a^{26} + 86a^{27} + a^{28}\big)$$

Green matrix

$$(aI + A)u = f$$

$$\Updownarrow$$

$$u = G(a)f$$

$$G(a) = (aI + A)^{-1} = \int_0^\infty e^{-at} H(t)dt$$

Pseudo Green matrix

$$E_0 f = 0 \quad \Longrightarrow$$

$$\begin{cases} Au = f \\ E_0 u = 0 \end{cases}$$

$$\Updownarrow$$

$$u = G_* f$$

$$G_* = \lim_{a \to +0} \left(G(a) - \frac{1}{a} E_0 \right)$$

Energy form

$$(u,\, v)_A \;=\; (Au,\, v) \;=\; v^* Au,$$

$$\| u \|_A^2 \;=\; (u,u)_A \;=\; E(u)$$

$$(u,\, v)_H \;=\; ((A+aI)u,\, v) \;=\; v^*(A+aI)u,$$

$$\| u \|_H^2 \;=\; (u,u)_H \;=\; E(a,u)$$

$$\delta_j = {}^t(0,\cdots,0,\underbrace{1}_{j},0,\cdots,0)$$

Reproducing relation

$$u \in \mathrm{C}^N \quad \text{and} \quad u(0) + u(1) + \cdots + u(N-1) = 0 \quad \Longrightarrow$$

$$u(j) = (u,\, G_* \delta_j)_A$$

$${}^t\delta_j G_* \delta_j = \| G_* \delta_j \|_A^2 = E(G_* \delta_j)$$

$$u \in \mathrm{C}^N \quad \Longrightarrow$$

$$u(j) = (u,\, G(a) \delta_j)_H$$

$${}^t\delta_j G_*(a) \delta_j = \| G(a) \delta_j \|_H^2 = E(a, G(a) \delta_j)$$