Himeji Conference on Partial Differential Equations
Titles and abstracts 2024

MINE, Takuya (Kyoto Institute of Technology)

Title: **Asymptotics of IDS for the random point interactions**

Abstract: In this talk, we give a survey about the asymptotics of the integrated density of states (IDS) at the bottom of the spectrum for Schrödinger operators with random point interactions. Especially, we prove that IDS for 2 or 3-dimensional Poisson point interactions decays polynomially at $-\infty$, and verify it by a numerical method.

TAGAWA, Tomoya (The University of Tokyo)

Title: **Low energy LAP for slowly decaying attractive potentials**

Abstract: In this talk, based on a joint work with Kenichi Ito, we discuss the uniform resolvent estimates near zero energy for the Schrödinger operator

$$H = -\tfrac{1}{2}\Delta + V + q \ \text{ on } \mathbb{R}^d$$

with $d \in \mathbb{N} = \{1, 2, \dots\}$. The potential V is *slowly decaying* and *attractive*, and q is a short-range perturbation relative to V. More precisely, we assume the following conditions. We use notation $\langle x \rangle = (1 + x^2)^{1/2}$.

Condition. Let $V \in C^2(\mathbb{R}^d)$ be spherically symmetric, and there exist $\nu, \epsilon \in (0, 2)$ and $c, C > 0$ such that for any $|\alpha| \le 2$ and $x \in \mathbb{R}^d$

$$|\partial^\alpha V(x)| \le C\langle x \rangle^{-\nu-|\alpha|}, \quad V(x) \le -c\langle x \rangle^{-\nu}, \quad x \cdot (\nabla V(x)) \le -(2-\epsilon)V(x).$$

In addition, let $q \in L^\infty(\mathbb{R}^d)$, and there exist $\nu' \in (\nu, 2]$ and $C' > 0$ such that for any $x \in \mathbb{R}^d$

$$|q(x)| \le C'\langle x \rangle^{-1-\nu'/2}.$$

Remark: Nakamura (1994), Fournais-Skibsted (2004), Richard (2006) and Skibsted (2013) discussed the LAP for C^∞ slowly decaying attractive potentials. We extend these previous results to C^2 potentials in the framework of an appropriate Agmon-Hörmander space without using microlocal analysis.

To begin with, we introduce Rellich's theorem which asserts the absence of generalized eigenfunctions with nonnegative eigenvalues in a certain weighted space called the Agmon-Hörmander spaces. Rellich's theorem is an important basis for our theory. Next, we will introduce the limiting absorption principle bounds and the radiation condition bounds. From these two estimates, we obtain the LAP. Finally, the Sommerfeld type uniqueness theorem, which characterizes the limiting resolvents, is introduced. In the proofs of our theorems, we mainly use a commutator argument recently invented by Ito-Skibsted.

BOURNE, Chris (Nagoya University)

Title: **Spectral and topological properties of quantum walks**

Abstract: Quantum walks, quantum analogues of random walks, bring together aspects of probability theory, quantum information theory and mathematical physics. In this talk, I will introduce an operator algebraic method to compute the essential spectrum of (discrete-time) quantum walk unitaries. For quantum walks with additional symmetries, we can also apply techniques from index theory to characterise 'topologically stable' bound states. This is partly based on arXiv:2211.10601 as well as work-in-progress with S. Richard and Y. Tanaka.

NATSUME, Toshikazu

Title: **A secondary index for certain non-Fredholm operators**

Abstract: We study a family of non-Fredholm operators. These operators are associated with the shift operator on the binary tree. We show that we can still extract numerical data so that operators are distinguished from each other. This is joint work with R. Nest.

BANERJEE, Rudrajit (Okinawa Institute of Science and Technology)

Title: **From Riemannian to Lorentzian manifolds: complex metrics and the Wick rotated heat semigroup**

Abstract: We introduce a 'Wick rotation', parameterized by $\theta \in [0, \pi/2]$, that interpolates between Riemannian and Lorentzian metrics on a smooth real manifold admitting codimension one foliations. Defined using the foliation structure, this interpolation maps between a Riemannian metric at $\theta = \pi/2$ and a Lorentzian metric at $\theta = 0$, passing through complex metrics at intermediate values of θ. We prove that for each $\theta \in (0, \pi/2]$, the associated Laplace-Beltrami operator generates an analytic semigroup, the 'Wick rotated heat semigroup', extending the usual heat semigroup arising from the Dirichlet Laplacian on a Riemannian manifold. In particular, for all $\theta \in (0, \pi/2]$: (i) the Wick rotated heat semigroup is smoothing on L^2, (ii) it has a smooth integral kernel, and (iii) the kernel's diagonal admits a small semigroup time asymptotic expansion. Finally, we show that (with an additional hypothesis) the Wick rotated heat semigroup converges to a unitary Schrödinger evolution group as $\theta \to 0$, i.e. in the strict Lorentzian limit.

This talk is based on joint work with Max Niedermaier at the University of Pittsburgh.

MIYAO, Tadahiro (Hokkaido University)

Title: **Stability of charge density waves in electron-phonon systems**

Abstract: Theoretical physicists have suggested that various types of order, such as superconducting and ferromagnetic states, emerge in many-electron systems on lattices. However, the mathematical justification for these phenomena is still not fully understood.

In this presentation, I will outline mathematical results concerning one of the typical orders in many-electron systems, namely, charge density waves. Firstly, I will briefly review the rigorous results on charge density waves in many-electron systems in two or more dimensions by C. Borgs and R. Kotecky. Subsequently, I will explain the stabilization of charge density waves in systems where electrons interact with lattice vibrations. This result aligns with predictions made by theoretical physicists using numerical computations.

The key points of the proof are as follows: 1. Extension of the Pirogov-Sinai theory by Borgs et al. for fermionic systems to systems where fermions interact with bosons. 2. Application of explicit formulas for correlation functions revealed in the rigorous study of quantum field theory to control contributions from bosons to the partition function.

NAKAHASHI, Wataru (Tokyo University of Science)

Title: **Non-Smoothness of the fundamental solutions for Schrödinger equations with super-quadratic and radially symmetric potentials**

Abstract: We consider the non-smoothness of the fundamental solution for the Schrödinger equation with a radially symmetric and super-quadratic potential. Yajima (1996) has shown that if the potential is super-quadratic and the spatial dimension is one, then the fundamental solution $E(t, x, y)$ is not smooth anywhere with respect to (t, x, y). Yajima has conjectured that the same result is true even for higher dimensional cases. In this talk, we show that the fundamental solution $E(t, x, y)$ does not belong to C^1 as a function of (t, x, y) if the potential is radially symmetric and super-quadratic, the spatial dimension is larger than or equals to 3. This talk is based on a joint work with Keiichi Kato (Tokyo University of Science) and Yukihide Tadano (Tokyo University of Science).

ISHIDA, Akari (Nagoya University)

Title: **A depth-dependent stability estimate in an iterative method for solving a Cauchy problem for the Laplace equation**

Abstract: In this talk, we consider the Cauchy problem for the Laplace operator. We construct approximate solutions by using the iterative method proposed by Bastay, Kozlov and Turesson. In this method, we solve the corresponding boundary value problems repeatedly. Then, we show that we construct them more stably when we choose the smaller domain where we consider the boundary value problems. Furthermore, since this method works with inexact data, we also deal with this case.

YOSHIDA, Naoya (Ritsumeikan University)

Title: **Trace asymptotics for the Iwatsuka Hamiltonian with strong electric field**

Abstract: In this talk, we consider the 2D magnetic Schrödinger operator

$$P = D_x^2 + (D_y - b(x))^2 + \mu V(x, y),$$

where V is a non-negative scalar potential decaying at infinity. The magnetic field is assumed to be positive, bounded and monotonically increasing on $\mathbb{R}$.

We obtain a complete asymptotic expansion of trf(P) as the coupling constant μ tends to $+\infty$, where $f \in C_0^\infty(\mathbb{R})$. We also give a Weyl type asymptotics formula with optimal reminder estimate of the counting function of eigenvalues of P.

This is based on the joint work with Profs. Shin-ichi Shirai (Osaka Institute of Technology).

PARK, Kanam (National Institute of Technology, Toba College)

Title: **Birational representations of affine Weyl groups arising from a Lax form of the q-Painlevé equation of type $E_6^{(1)}$**

Abstract: The q-Painlevé equation of type $E_6^{(1)}$ is given as a q-difference system with an affine Weyl group symmetry of type $E_6^{(1)}$. We obtained a 3×3 Lax formalism for the equation as a new one. Our goal is to understand a birational representation of an affine Weyl group symmetry of type $E_6^{(1)}$ as actions on the 3×3 Lax formalism. In this talk, we talk about a relation between it and known birational actions of 3 kinds of affine Weyl group of type $A_2^{(1)}$ as a partial result.

MASAKI, Satoshi (Hokkaido University)

Title: **Global existence and asymptotic behavior in nonlinear Schrödinger systems without null and dissipative structure**

Abstract: We consider a class of systems of cubic nonlinear Schrödinger equations with two components in one dimension. It is known that cubic nonlinearity is critical in one dimension in view of the large-time behavior. (cf. modified scattering). Compared to the single-equation case, systems exhibit a wide range of asymptotic behaviors for their solutions. Our motivation is to comprehend the effect of general nonlinearities that are gauge-invariant and represented by cubic polynomials with real coefficients.

For systems within this class, the global existence of small solutions is not always guaranteed. One sufficient condition for global existence is the presence of a conserved quantity equivalent to the $(L^2)^2$-norm of the solution. In previous studies of large-time behavior, the assumption of the existence of such a conserved quantity is often made, characterized as a so-called weak null condition. Another sufficient condition for global existence is the validity of a dissipative condition, indicating the non-increasing property of a quantity equivalent to the $(L^2)^2$-norm of the solution.

In this talk, we introduce a specific class of systems that do not satisfy the weak null condition or the dissipative condition, and that, however, we can establish the global existence of solutions for small data. We also provide an asymptotic profile of such solutions in terms of a solution to the corresponding ODE system.

Organized by : HIGUCHI, K., HIROTA, K., ITAKURA, K., KAMEOKA, K., NAGAYASU, S., RICHARD, S., TAIRA, K., WATANABE, T., YOSHIDA, N.