Himeji Conference on Partial Differential Equations
Titles and abstracts 2025

ISHIDA, Atsuhide (Tokyo University of Science)

Title: **Stummel-type perturbations for fractional and relativistic Schrödinger operators**

Abstract: In this talk, we propose new classes of potential functions as perturbations of fractional and relativistic Schrödinger operators. Specifically, we define four types of fractional Stummel classes that ensure infinitesimal relative boundedness and relative compactness, based on the Stummel conditions for the standard Schrödinger operator. If time permits, we will also discuss fractional Kato classes. This is joint work with J. Lőrinczi and G. Ascione.

HOSHIYA, Akitoshi (The University of Tokyo)

Title: **Semiclassical limit of orthonormal Strichartz estimates on scattering manifolds**

Abstract: We consider the orthonormal Strichartz estimates on scattering manifolds. Under the nontrapping condition we can prove global-in-time estimates for the same exponent as in the Euclidean space. We show that these estimates imply the global-in-time Strichartz estimates for kinetic transport equations in the semiclassical limit. As byproducts we obtain sufficient conditions on trapped sets that break the orthonormal Strichartz estimates without constructing quasimodes. An application to the scattering for the Boltzmann equation will be also discussed.

PARRA, Daniel (University of La Frontera)

Title: **Eigenvalue Asymptotics near a flat band**

Abstract: In this talk we start by briefly discussing the existence of flat bands for discrete periodic structures. Then, we will focus our attention to a Dirac-type operator on $\mathbb{Z}^n$ perturbed by a multiplication operators by a slowly decaying function. We show that the eigenvalues accumulate near the value of the flat band at a "semiclassical" rate with a constant that encodes the structure of the flat band.

KAWAMOTO, Masaki (Okayama University)

Title: **Global well-posedness for nonlinear Schrödinger equations below the Strauss exponent**

Abstract: In this paper, we consider the nonlinear Schrödinger equation with a general homogeneous nonlinearity in dimensions up to three. We establish global well-posedness (GWP) and scattering for small data in the standard weighted space for a class of homogeneous nonlinearities, including non-gauge-invariant ones. Additionally, we include the case where the degree is less than or equal to the Strauss exponent. We prove GWP by rewriting the conventional integral equation into a new form in which the resolvent of the harmonic oscillator appears. This talk is based on joint work with Masaki Satoshi (Hokkaido Univ.) and Hayato Miyazaki (Kagawa Univ).

SAKAMOTO, Shota (Kyushu University)

Title: **Global solutions to the non cutoff Boltzmann equation with low-regularity initial data**

Abstract: We will consider the Cauchy problem of the non cutoff Boltzmann equation in a so-called low regularity space. This space is characterized by the integrability of the Fourier transform of a function, and this condition requires much weaker spatial regularity on initial data than those previously used for the study of the Cauchy problem. We establish the existence and uniqueness of global solutions to the Boltzmann equation on the torus and Euclidean spaces around the global Maxwellian equilibrium. If time allows, we will show some decay estimates of them. This is based on joint works with R.-J. Duan, S.-Q. Liu, R. M. Strain, and Y. Ueda.

GELL-REDMAN, Jesse (The University of Melbourne)

Title: **The Feynman propagator for the Klein-Gordon equation**

Abstract: We construct the Feynman propagator for Klein-Gordon (KG) equation on Minkowski space perturbed by a decaying spatial potential. In particular, we construct global in time solutions to the inhomogeneous KG equation, each of whose wavefront sets is contained in the flowout of the wavefront set of the source in the direction of the Hamilton flow. Such solutions were shown to exist locally by Duistermaat-Hörmander. To accomplish this, we prove a global Fredholm estimate. The persistence of the potential in time means that estimates for KG can be obtained using positive commutator estimates with operators in the three-body calculus of Vasy. This is joint work with Dean Baskin and Moritz Doll.

LOUATRON, Vincent (Ritsumeikan University)

Title: **Resonances for matrix-valued Schrödinger operators via normal forms**

Abstract: We study the semiclassical distribution of resonances of 2×2, 1D matrix-valued Schrödinger operators such as

$$\mathscr{P} := \begin{pmatrix} -h^2\frac{\mathrm{d}^2}{\mathrm{d}x^2} + V_1(x) & hW(x) \\ hW(x) & -h^2\frac{\mathrm{d}^2}{\mathrm{d}x^2} + V_2(x) \end{pmatrix}$$

where the potentials V_j and the interaction term W are (multiplication operators by) smooth, real-valued functions. Here $h > 0$ denotes the semiclassical parameter. Such operators arise in quantum chemistry: in the Born-Oppenheimer approximation, the reciprocal of the imaginary part of resonances describes the half-life of quantum particles.

The semiclassical distribution of resonances is governed by the geometry of the underlying classical mechanics of $\mathscr{P}$. When the classical trajectories associated to $\mathscr{P}$ cross, the semiclassical asymptotics of this distribution are described by a *microlocal transfer matrix* at the crossing points. It is precisely computed using a stationary phase method.

In this talk, we discuss a generalization of the existing normal form reductions for transversal crossings to the case of tangential crossings, based on a joint work with K.Higuchi and K.Taira. This method reduces the computation of the microlocal transfer matrix of a general pseudo-differential operator (including the Schrödinger type) to that of

$$P := \begin{pmatrix} h\frac{\mathrm{d}}{\mathrm{d}x} & hR_1 \\ hR_2 & h\frac{\mathrm{d}}{\mathrm{d}x} - f(x) \end{pmatrix}$$

with suitable R_j and a smooth function f describing the geometry of the crossing point.

KATO, Koichi (Nagoya University)

Title: **Topological crystals: Independence of spectral properties with respect to reference systems**

Abstract: It is a common postulate that spectral properties of operators describing physical systems are independent of the underlying reference frames. During this talk and for the Laplace operator on topological crystals, we shall prove such a statement from a deeper analysis of the behavior of spectral properties with respect to arbitrary choices. In particular, we shall present the impact of the choice of a unit cell, and of the choice of a family of generators for the transformation group.

KAMIMOTO, Shingo (Kanazawa University)

Title: **Resurgence of WKB solutions**

Abstract: The resurgence of WKB solutions is one of the important open problems in the exact WKB analysis. In this talk, we give recent progresses on this problem for the case of polynomial potentials. This talk is based on a joint work with David Sauzin (CNRS).

TAKIZAWA, Shun (Tokyo University of Science)

Title: **Boundedness of propagators for Dirac equations with potentials on Wiener amalgam spaces**

Abstract: We consider boundedness of propagators for Dirac equations on Wiener amalgam spaces. The Wiener amalgam space is a function space that separately measures the local singularities of a function and its decay at infinity. In the case where the potential is bounded, Trapasso (2020) has studied boundedness of propagators for Dirac equations. The purpose of this talk is to prove boundedness of propagators for Dirac equations with unbounded time-dependent potentials. In particular we deal with a class of potentials, including Stark and harmonic potentials.

MIZUTANI, Haruya (Osaka University)

Title: **Uniform resolvent and smoothing estimates for the Heisenberg sublaplacian**

Abstract: We discuss a recent progress (arXiv:2409.11943) on uniform resolvent estimates and smoothing effects for the sublaplacian on the Heisenberg group. The Schrödinger equation with the Heisenberg sublaplacian is known to be a totally non-dispersive equation since it has soliton solutions with finite speed of propagation. In particular, usual global-in-time dispersive and Strichartz estimates cannot hold. Nevertheless, we can show some uniform weighted resolvent estimates without loss of derivatives. The proof is based on the method of weakly conjugate operators with the generator of an anisotropic dilation as a conjugate operator. By the method of smooth perturbations, we also obtain local smoothing effects for the associated Schrödinger equation. This is joint work with Luca Fanelli, Luz Roncal and Nico Michele Schiavone.

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