

Influence of virtual turning points upon non-adiabatic transitions in three-level systems and related problems of the exact WKB analysis

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In this talk, we consider non-adiabatic transition probabilities of 3-level systems from the viewpoint of the exact WKB analysis ([AKT]). Namely we study the exact WKB analysis of a system

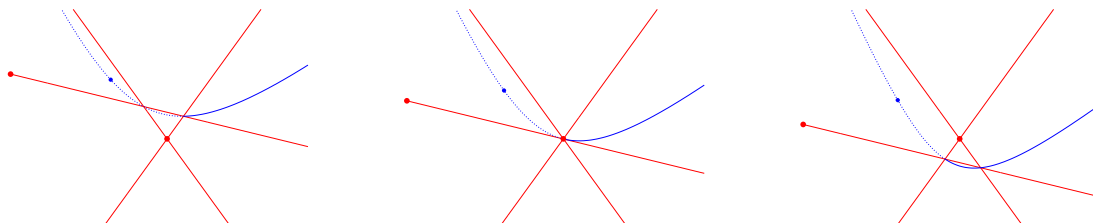
$$i\frac{d}{dt}\Psi = \eta H(t, \eta)\Psi \quad (1)$$

with a large parameter $\eta > 0$. Here $\Psi = \Psi(t, \eta)$ is a 3-vector and $H(t, \eta)$ is a 3×3 matrix given below:

$$\begin{aligned} H(t, \eta) &= H_0(t) + \eta^{-1/2} H_{1/2} \\ &= \begin{pmatrix} \rho_1(t) & 0 & 0 \\ 0 & \rho_2(t) & 0 \\ 0 & 0 & \rho_3(t) \end{pmatrix} + \eta^{-1/2} \begin{pmatrix} 0 & c_{12} & c_{13} \\ \bar{c}_{12} & 0 & c_{23} \\ \bar{c}_{13} & \bar{c}_{23} & 0 \end{pmatrix} \end{aligned}$$

with (real) polynomials $\rho_1(t)$, $\rho_2(t)$, $\rho_3(t)$ and complex constants c_{12} , c_{13} , c_{23} . Following [Sh] and [Sa1], we focus on the influence of virtual turning points (VTPs for short) and new Stokes curves (NSCs) (See [HKT] for definitions.) upon transition probabilities, and demonstrate that VTPs and NSCs can be important in practical application.

We also discuss some basic problems of the exact WKB analysis related to the present theme. Especially we study the situation in which a Stokes curve hits a double turning point of different type.



References

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