Some appplications of Carleman estimates for parabolic operators in an isotropic discontinuous media

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Abstract

We consider

- the cylinder $\Omega := (0,1) \times (-H, H), \Omega_+ := (0,1) \times (0, H), \Omega_- := (0,1) \times (-H, 0)$
- a real diffusion coefficient a such that the restrictions $a_\pm$ to $\Omega_\pm$ belong to $\mathcal{C}^2(\bar{\Omega}_\pm)$ and $0 < c_{\min} \leq a \leq c_{\max} < +\infty$.
- the positive selfadjoint operator $A = -\nabla \cdot (a\nabla)$ with Dirichlet boundary condition.

So, the discontinuities of the coefficient a are included in the interface $S = (0,1) \times \{0\}$. We prove a Carleman estimate for the heat operator $\partial_t + A$ on $\Omega \times (0,T)$ when the localization of the observation is on the boundary of $\partial\Omega$. Precisely, we prove

Theorem 1. *There exist a weight function $\varphi(x,t) := e^{d(x)-\beta(t-t_0)^2}$ and positive constants C, s_0, λ_0 such that, for $s > s_0$ and $\lambda > \lambda_0$, we have*

$$C \int_{\Omega\times(0,T)} \Big(e^{2s\varphi}(s^3\lambda^4\varphi^3|u|^2 + s\lambda^2\varphi|\nabla u|^2 + \frac{1}{s\varphi}|\partial_t u|^2\Big)\mathrm{d}x\mathrm{d}t$$
$$+ \int_{S\times(0,T)} s\lambda\varphi e^{2s\varphi}\big(|(\nabla u)_{|S_i\times(0,T)}|^2 + (s\lambda)^2\varphi^2|u|^2\big)\mathrm{d}\gamma_S\mathrm{d}t - \int_{\partial\Omega\times(0,T)} e^{2s\varphi}s\lambda\varphi a^2(\partial_\nu d)(\partial_\nu u)^2\mathrm{d}\gamma\mathrm{d}t$$
$$\leq \int_{\Omega\times(0,T)} f^2 e^{2s\varphi}\mathrm{d}x\mathrm{d}t,$$

for $u(\cdot,t) \in D(A)$ and $u(\cdot,0) = u(\cdot,T) = 0$.

This kind of inequality for a parabolic operator is well known when the diffusion coefficient is regular or when the interface S is far from the boundary[1]. Here, the interface is tranverse to the boundary and the weight function φ is not usual for a parabolic operator. Starting from this estimate, we shall develop applications that estimate the restriction of u on some subsets of $\Omega \times (0,T)$ from the knowledge of the normal derivative of u on a part of $\partial\Omega$.

[1] Le Rousseau J. & Robbiano L., Local and global Carleman estimates for parabolic operators with coefficients with jumps at interfaces. Inv. math (2011) 183:245-336.