Abstract

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"Real eigenvalues of a non-self-adjoint perturbation of the defocusing Zakharov-Shabat operator"

We study the eigenvalue problem for the operator

$$L = \begin{pmatrix} ih\dfrac{d}{dx} & -iA(x) \\ iA(x) & -ih\dfrac{d}{dx} \end{pmatrix}.$$

Here, h is a positive small parameter, and $A(x)$ is a real-valued potential. This operator is called Zakharov-Shabat operator which is one of the Lax pair in the inverse scattering method for the defocusing nonlinear Schrödinger equation. It is self-adjoint, and has real eigenvalues when $A(x)^2$ has a well. In particular, we consider the case where $A(x)^2$ has a simple well, and derive the Bohr-Sommerfeld type quantization condition for the eigenvalues.

Next, we add a small perturbation to the operator L:

$$L_\varepsilon = \begin{pmatrix} ih\dfrac{d}{dx} & -i\left(A(x) + i\varepsilon B(x)\right) \\ i\left(A(x) + i\varepsilon B(x)\right) & -ih\dfrac{d}{dx} \end{pmatrix},$$

where ε is a positive small parameter, and $B(x)$ is a real-valued function. This operator is no longer self-adjoint, and the eigenvalues become complex in general. However, we may expect that the eigenvalues stay real when $A(x)$ and $B(x)$ have some symmetry, as in the case of the Schrödinger operator with $\mathcal{PT}$-symmetry. Our aim is to show that the eigenvalues of L_ε are real when $A(x)$ is even and $B(x)$ is odd, or $A(x)$ is odd and $B(x)$ is even, with a sufficiently small h and ε.