Generalization of the Maxwell Equation and First-order Expression of the Elastic Equation

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First of all, we explain that the Maxwell equation (in the vacuum)

$$\begin{cases} \partial_t(\varepsilon\vec{E}) = \text{curl } \vec{H}, \\ \partial_t(\mu\vec{H}) = -\text{curl } \vec{E} \end{cases}$$

can be transformed into the elastic equation of the transversal waves

$$\rho\ \partial_t^2 u = \tilde{\mu} \text{ curl (curl } u).$$

Next, we introduce some known generalizations of the Maxwell equation by physists so as to admit existence of the longitudinal waves together with the transversal ones.

Finally, we propose a first-order hyperbolic system integrating the above generalized Maxwell equations:

$$D_t \begin{pmatrix} v \\ \tilde{v} \end{pmatrix} - \begin{pmatrix} A(D_x) & B(D_x) \\ -B(D_x) & \tilde{A}(D_x) \end{pmatrix} \begin{pmatrix} v \\ \tilde{v} \end{pmatrix} = 0.$$

And we show that this system is equivalent to a (generalized) elastic equation through some natural transformation:

$$\left\{ D_t^2 - (A(D_x) + \tilde{A}(D_x))D_t + A(D_x)\tilde{A}(D_x) + B(D_x)^2 \right\} u = 0.$$

Our discussions are based on the analysis of the linear algebra for the symbol.