Lectures on Semi-Classical Analysis 2024

Dates: October 28 (Mon) – October 30 (Wed), 2024

Place: Colloquium Room, West Wing, 6th floor, BKC, Ritsumeikan University

Program

October 28 (Monday)

10:00–10:50 MARTINEZ, André (University of Bologna)
The microsupport of functions and applications I

10:50–11:30 Discussion

13:00–13:50 BEHRNDT, Jussi (Technische Universität Graz)
The Landau Hamiltonian with delta-potentials supported on curves

13:50–14:20 Discussion

14:20–15:10 TAIRA, Kouichi (Kyushu University)
The WKB method and its application to spectral theory

15:10–15:40 Discussion

15:40–16:05 JI, Yong-Gwan (Korea Institute for Advanced Study)
Finiteness of the stress in presence of closely located inclusions with imperfect bonding

18:00– Banquet

October 29 (Tuesday)

10:00–10:50 MARTINEZ, André (University of Bologna)
The microsupport of functions and applications II

10:50–11:30 Discussion

13:00–13:50 NAKAMURA, Shu (Gakushuin University)
Microlocal methods in scattering theory I. Microlocal resolvent estimates.

13:50–14:20 Discussion

14:20–15:10 HIGUCHI, Kenta (Ehime University)
Propagation of wave packets and its applications

15:10–15:40 Discussion

15:40–16:30 TAGAWA, Tomoya (The University of Tokyo)
Uniform limiting absorption principle for slowly decaying attractive potentials

FUKUSHIMA, Shota (Chiba Institute of Technology)
On localized plasmons on axially symmetric domains

16:30– Poster / Aperitif

HOSOKAWA, Sota; ITO, Masato; KAWAMURA, Yuta; LOUATRON, Vincent;

October 30 (Wednesday)

10:00–10:50 MARTINEZ, André (University of Bologna)
The microsupport of functions and applications III

10:50–11:30 Discussion

13:00–13:50 HOSHIYA, Akitoshi (The University of Tokyo)
Resolvent estimates for Schrödinger operator with quadratic repulsive potentials

MURAMATSU, Ryo (Tokyo University of Science)
Wellposedness in the modulation spaces of the Schrödinger equation with the vector potential in the Sjöstrand class

14:10–15:00 NAKAMURA, Shu (Gakusuin University)
Microlocal methods in scattering theory II. Scattering matrices as a quantization of the classical scattering map.

15:00–15:30 Discussion

Abstract of the courses

Lecturer: MARTINEZ, André

Title: The microsupport of functions and applications

Abstact of Lecture 1: Short review of the semiclassical Fourier transform. The Fourier-Bros-Iagolnitzer transform and the microsupport of functions. Basic properties.

Abstact of Lecture 2: The microsupport of solutions to analytic linear PDE's: The characteristic set. Propagation along the classical Hamilton flow. Microhyperpolicity. The Kawai-Kashiwara theorem.

Abstact of Lecture 3: Extension to systems of PDE's. A case of propagation through a non involutive crossing of characteristic sets. Application to the predissociation of molecules.

REFERENCES:

Lectures 1 and 2: the arguments are included in my book " An Introduction to Semiclassical and Microlocal Analysis " (Springer, 2000).

Lecture 3: the main result is extracted from the article "Widths of highly excited resonances in multidimensional molecular predissociation " by Vania Sordoni and myself (J. Math. Soc. Japan 72(3): 687-730, July, 2020). It consists of a generalization of Proposition 7.2 (and its proof) in this article.

For a deeper insight into the analytic microlocal analysis one may consult the book " Singularités Analytiques Microlocales " by Johannes Sjöstrand (Astérisque n. 95, 1982).

Lecturer: NAKAMURA, Shu

Title: 1. Title: Microlocal methods in scattering theory. I. Microlocal resolvent estimates.

Abstract: We explain that the microlocal resolvent estimates due to Isozaki and Kitada follow from the location of the wave front set of the resolvent in the Fourier space. The proof employs a propagation estimate analogous to the one used in the proof of the microlocal smoothing estimates.

Title: 2. Title: Microlocal methods in scattering theory. II. Scattering matrices as a quantization of the classical scattering map.

Abstract: We construct a classical scattering theory for Schrödinger operator with long-range type perturbations, and show that the quantum scattering matrix is a quantization of the classical scattering map in terms of the Fourier integral operator method. Here we employ the Isozaki-Kitada time-independent modifiers to define the wave operators.

Lecturer: TAIRA, Kouichi

Title: The WKB method and its application to spectral theory

Abstract: The purpose of this lecture is to explain the basics of the WKB method and its application to spectral theory. First, we introduce the definition of Lagrangian distributions, also known as WKB states. Second, we provide some calculations involving Lagrangian distributions. Finally, we characterize the existence of discrete eigenvalues using Bohr-Sommerfeld conditions for a class of one-dimensional Schrödinger operators.

Lecturer: HIGUCHI, Kenta

Title: Propagation of wave packets and its applications

Abstract: In this lecture, we study the Cauchy problem for the Schrödinger equation with an initial state taken as a wave packet. We consider wave packets whose position and momentum are localized in the semiclassical limit. Our main objective is to demonstrate that the centers of the position and momentum of the propagated state obey the corresponding classical mechanics. This fact applies to certain spectral and scattering problems.

Abstract of the talk

Speaker: BEHRNDT, Jussi

Title: The Landau Hamiltonian with delta-potentials supported on curves

Abstract: In this talk we investigate the spectral properties of a singularly perturbed self-adjoint Landau Hamiltonian with a delta-potential supported on a finite curve. After a general discussion of the qualitative spectral properties of the operator and its resolvent, one of our main objectives is the local spectral analysis near the Landau levels. Under various conditions it will be shown that the perturbation smears the Landau levels into eigenvalue clusters, and the accumulation rate of the eigenvalues within these clusters is determined. This talk is based on joint papers with P. Exner, M. Holzmann, V. Lotoreichik, and G. Raikov.

Speaker: JI, Yong-Gwan

Title: Finiteness of the stress in presence of closely located inclusions with imperfect bonding

Abstract: If two conducting or insulating inclusions are closely located, the gradient of the solution may become arbitrarily large as the distance between inclusions tends to zero, resulting in high concentration of stress in between two inclusions. This happens if the bonding of the inclusions and the matrix is perfect, meaning that the potential and flux are continuous across the interface. In this paper, we consider the case when the bonding is imperfect. We consider the case when there are two circular inclusions of the same radii with the imperfect bonding interfaces and prove that the gradient of the solution is bounded regardless of the distance between inclusions if the bonding parameter is finite. This result is of particular importance since the imperfect bonding interface condition is an approximation of the membrane structure of biological inclusions such as biological cells. This is joint work with Shota Fukushima, Hyeonbae Kang, and Xiaofei Li.

Tittle of Poster session

- HOSOKAWA, Sota;
  WKB construction at a potential minimum and its applications

- ITO, Masato;
  Eigenvalue splitting for a piecewise linear double well and a numerical simulation

- KAWAMURA, Yuta;
  Propagation of the frequency set for matrix-valued pseudo-differential operators

- LOUATRON, Vincent;
  Normal forms for 2×2 *matrix-valued operators and applications*

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