# CONTROL FOR SCHRÖDINGER OPERATORS ON 2-TORI: ROUGH POTENTIALS

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Abstract. For the Schrödinger equation, $(i\partial_t + \Delta + V)u = 0$ on a torus, an arbitrary non-empty open set $\Omega$ provides observability of the solution: $\|u|_{t=0}\|_{L^2(T^2)} \leq K_T \|u\|_{L^2((0,T)\times\Omega)}$. As a consequence, one can classically deduce the following control result

**Theorem 1.** *Let $\Omega \subset T^2$ be any nonempty open set and let $T > 0$. For any $u_0 \in L^2(T^2)$, there exists $f \in L^2((0,T)\times\Omega)$ such that the solution of the equation*

$$(i\partial_t + \Delta - V(z))u(t,z) = f \mathbb{1}_{(0,T)\times\Omega}(t,z), \qquad u(0,\bullet) = u_0,$$

*satisfies*

$$u(T,\bullet) \equiv 0.$$

This result was first obtained by Jaffard ($V = 0$), Anantharaman-Macia, and Burq-Zworski ($V \in C(\mathbb{T}^2)$) and conjectured for $V \in L^\infty(\mathbb{T}^2)$. Here we prove the result for $V \in L^2(T^2)$ (note that at that level of regularity the group evolution is still well defined). The higher dimensional generalization remains open for $V \in L^\infty(\mathbb{T}^n)$. These results were obtained in collaboration with J. Bourgain and M. Zworski.

Plan of the course:

- Course 1. "Control Theory" In this course, I will present the general framework of control theory using basic functional analysis (HUM method, observation) and also the basic tools of semi-classical analysis required in the course
- Course 2. "Dispersion on tori" In this course, I will present some dispersive type estimates enjoyed by solutions of SchrAvdinger equations on tori. I will also show how these estimates allow to get a better understanding of the operator groups $e^{it(\Delta+V)}$ on $L^2(T^2)$.
- Course 3. In this course, I will precise the ingredients for the proof of the control result.

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