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Title:

"Spectral properties of semiclassical non-selfadjoint operators"

Abstract:

Non-selfadjoint operators appear naturally in a number of areas, from solvability theory for linear PDE to quantum mechanics to kinetic theory. An essential feature of their spectral analysis is that for such operators, the spectrum does not control the resolvent, which may become very large even far from the spectrum. This creates a challenge, but also an opportunity, accounting for some of the complex and fascinating traits in the spectral behavior of non-selfadjoint operators.

Powerful tools of analytic microlocal analysis become available in the case of non-selfadjoint differential operators with analytic coefficients, and in this case it turns out that the spectrum is often determined by the behavior of the holomorphic continuation of the symbol along suitable complex deformations of the real phase space. The purpose of this mini-course is to provide an overview of some of the recent developments in the non-selfadjoint spectral theory, with an emphasis on the analytic setting, and to attempt to illustrate some of its inner workings. The tentative plan of the lectures is as follows:

Lecture 1. Semiclassical non-selfadjoint pseudodifferential operators, their spectra and semiclassical pseudospectra. Absence of eigenvalues and resolvent bounds close to the boundary of the pseudospectrum under a non-trapping condition. Harmonic approximation for non-selfadjoint operators and spectral asymptotics for the low-lying eigenvalues. Hörmander-Davies quasimodes and spectral instability for non-normal semiclassical operators.

Lecture 2. Non-selfadjoint pseudodifferential operators with holomorphic symbols. FBI-Bargmann transformations and pseudodifferential operators on the FBI transform side. Phase space exponential weights and complex deformations of the real cotangent space. Method of averaging for analytic non-selfadjoint operators. Bohr-Sommerfeld quantization conditions for non-selfadjoint analytic operators in dimension one.

Lecture 3. Non-selfadjoint perturbations of analytic selfadjoint operators with completely integrable classical flows in dimension two. Microlocal normal forms near Diophantine Lagrangian tori and spectral asymptotics. Extensions to KAM classical flows. Weyl laws for the distribution of the imaginary parts of the eigenvalues. Numerical illustrations in 2D and the role of rational invariant tori.