RIMS 共同研究（公開型）

幾何構造と微分方程式

ー 対称性・特異点及び量子化の視点から ー

Geometric Structures and Differential Equations – Symmetry, Singularity, and Quantization –

Program and Abstracts

14 – 18 March 2022

Zoom meeting/
Ritsumeikan University
Biwako-Kusatsu Campus
Westwing 6F Colloquium room

Organizers:
Daisuke Tarama (多羅間 大輔, Ritsumeikan University)
Kenro Furutani (古谷 賢朗, Osaka City University / Tokyo University of Science)
Hiroaki Yoshimura (吉村 浩明, Waseda University)

Time Schedule

Remark: The following schedule is written in
Japanese Standard Time (JST = UTC + 0900) / Central European Time (CET = UTC + 0100).

March 14 (Monday)		
12:50 – 13:00 (JST)	4:50 – 5:00 (CET)	Opening
13:00 – 14:00 (JST)	5:00 – 6:00 (CET)	01 Takuma Tomihisa
14:20 – 15:20 (JST)	6:20 – 7:20 (CET)	02 Hidekazu Ito
16:00 – 17:00 (JST)	8:00 – 9:00 (CET)	03 Jean-Pierre Françoise
17:20 – 18:20 (JST)	9:20 – 10:20 (CET)	04 Sonja Hohloch
March 15 (Tuesday)		
13:00 – 14:00 (JST)	5:00 – 6:00 (CET)	05 Akira Kitaoka
14:20 – 15:20 (JST)	6:20 – 7:20 (CET)	06 Kazuyuki Yagasaki
16:00 – 17:00 (JST)	8:00 – 9:00 (CET)	07 Tudor S. Ratiu
17:20 – 18:20 (JST)	9:20 – 10:20 (CET)	08 Wolfram Bauer
March 16 (Wednesday)		
13:00 – 14:00 (JST)	5:00 – 6:00 (CET)	09 Yuji Hirota
14:20 – 15:20 (JST)	6:20 – 7:20 (CET)	10 Tomoyuki Kakehi
16:00 – 17:00 (JST)	8:00 – 9:00 (CET)	11 Elmar Schrohe
17:20 – 18:20 (JST)	9:20 – 10:20 (CET)	12 Abdellah Laaroussi
March 17 (Thursday)		
13:00 – 14:00 (JST)	5:00 – 6:00 (CET)	13 Shoya Motonaga
14:20 – 15:20 (JST)	6:20 – 7:20 (CET)	14 Hiroyuki Chihara
16:00 – 17:00 (JST)	8:00 – 9:00 (CET)	15 Ryszard Nest
17:20 – 18:20 (JST)	9:20 – 10:20 (CET)	16 Irina Markina
March 18 (Friday)		
13:00 – 14:00 (JST)	5:00 – 6:00 (CET)	17 Genki Ishikawa
14:20 – 15:20 (JST)	6:20 – 7:20 (CET)	18 Setsuro Fujiié
15:20 – 15:30 (JST)	7:20 – 7:30 (CET)	Closing

(0) Basic Information

Dates: From 14/March/2022 till 18/March/2022

Participation: Through Zoom meeting.

Please register through the following URL:
`https://ritsumei-ac-jp.zoom.us/meeting/register/tJYpfuyoqDsoGd0OoIvjRCGAW494F6ES2TJT`

Web-site:
`http://www.math.ritsumei.ac.jp/~dtarama/GSDE2022/index.html`

(1) Program

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Japanese Standard Time (JST = UTC + 0900) / Central European Time (CET = UTC + 0100).

Monday, 14/March/2022

12:50 – 13:00 (JST) / 4:50 – 5:00 (CET) Opening

13:00 – 14:00 (JST) / 5:00 – 6:00 (CET) Takuma Tomihisa (Waseda University)
Higher spin Dirac operators

14:20 – 15:20 (JST) / 6:20 – 7:20 (CET) Hidekazu Ito (Kanagawa University)
Birkhoff normalization for a family of superintegrable symplectic maps and its application

16:00 – 17:00 (JST) / 8:00 – 9:00 (CET) Jean-Pierre Françoise (Sorbonne University)
Integrable Systems and their discretization

17:20 – 18:20 (JST) / 9:20 – 10:20 (CET) Sonja Hohloch (University of Antwerp)
S^1*-actions, semitoric systems, and hyperbolic singularities*

Tuesday, 15/March/2022

13:00 – 14:00 (JST) / 5:00 – 6:00 (CET) Akira Kitaoka (University of Tokyo)
Ray-Singer torsion and the Rumin Laplacian on lens spaces

14:20 – 15:20 (JST) / 6:20 – 7:20 (CET) Kazuyuki Yagasaki (Kyoto University)
Some recent results on nonintegrability of dynamical systems

16:00 – 17:00 (JST) / 8:00 – 9:00 (CET) Tudor S. Ratiu (Shanghai Jiao Tong University)
Teichmüller space and differential character valued momentum maps

17:20 – 18:20 (JST) / 9:20 – 10:20 (CET) Wolfram Bauer (Leibniz University Hannover)
Trivializable subriemannian structures on spheres and subelliptic heat equation

Wednesday, 16/March/2022

13:00 – 14:00 (JST) / 5:00 – 6:00 (CET) Yuji Hirota (Azabu University)
Dirac structures over the space of connections on 3 and 4 dimensional Riemannian manifolds

14:20 – 15:20 (JST) / 6:20 – 7:20 (CET) Tomoyuki Kakehi (University of Tsukuba)
Surjectivity of convolution operators

16:00 – 17:00 (JST) / 8:00 – 9:00 (CET) Elmar Schrohe (Leibniz University Hannover)
Degenerate Elliptic Boundary Value Problems with Non-smooth Coefficients

17:20 – 18:20 (JST) / 9:20 – 10:20 (CET) Abdellah Laaroussi (Leibniz University Hannover)
Heat kernel asymptotics for quaternionic contact manifolds

Thursday, 17/March/2022

13:00 – 14:00 (JST) / 5:00 – 6:00 (CET) Shoya Motonaga (Kyoto University)
Obstructions to integrability of nearly integrable dynamical systems near regular level sets

14:20 – 15:20 (JST) / 6:20 – 7:20 (CET) Hiroyuki Chihara (University of the Ryukyus)
Microlocal analysis of d-plane transform on the Euclidean space

16:00 – 17:00 (JST) / 8:00 – 9:00 (CET) Ryszard Nest (University of Copenhagen)
Automorphisms of Boutet de Monvel algebra

17:20 – 18:20 (JST) / 9:20 – 10:20 (CET) Irina Markina (University of Bergen)
A unified approach to extremal curves on Stiefel manifolds

Friday, 18/March/2022

13:00 – 14:00 (JST) / 5:00 – 6:00 (CET) Genki Ishikawa (Ritsumeikan University)
An elliptic fibration arising from the Lagrange top

14:20 – 15:20 (JST) / 6:20 – 7:20 (CET) Setsuro Fujiié (Ritsumeikan University)
WKB and microlocal approach to various Bohr-Sommerfeld quantization rules

15:20 – 15:30 (JST) / 7:20 – 7:30 (CET) Closing

(2) Abstracts

Title: Higher spin Dirac operators

Takuma Tomihisa
Waseda University, Japan

Abstract:
Higher spin Dirac operators are generalizations of the Dirac operator and the Rarita-Schwinger operator. They are geometric first order differential operators on the spinor fields with spin $j + 1/2$ over Riemannian spin manifolds. In this talk, I will introduce higher spin Dirac operators and give some formulas for higher spin Dirac operators on spaces of constant curvature. Furthermore, we calculate spectra of higher spin Dirac operators on the sphere by using formulas. This gives Branson's result (2002) in a different way of calculation. This talk is based on a joint work with Yasushi Homma (Waseda University).

Title: Birkhoff normalization for a family of superintegrable symplectic maps and its application

Hidekazu Ito
Kanagawa University, Japan

Abstract:
Integrability of a Hamiltonian system with n degrees of freedom is defined by existence of n Poisson-commuting integrals. The Liouville-Arnold's theorem states that, if the level set of those integrals is regular, compact and connected, then it is an n dimensional torus and there exists a system of special symplectic coordinates called "action-angle variables" in which the system can be solved explicitly.
The aim of this talk is to clarify the orbit structure of an (super-)integrable system near singularities of those integrals. The special feature is that the singularities are resonant in the sense of dynamical systems. For this purpose, we focus on Birkhoff normalization for a family of $2n$ dimensional analytic symplectic maps near resonant fixed points and prove the existence of a convergent Birkhoff transformation provided that the maps have $n + q$ integrals near a fixed point of resonance degree q. It leads to the existence of a special coordinate system in which the given Hamiltonian system can be solved explicitly near singularities of its integrals.

Title: Integrable Systems and their discretization

Jean-Pierre Françoise
Sorbonne University, France

Abstract:
We consider discretizations of integrable systems in dimension two, which are defined by birational maps. We recall that in the case of two dimensional cubic planar vector fields, such discretizations are associated with automorphisms of elliptic fibrations. In contrast, the discrete Lotka-Volterra system integrability is not yet established. This is the same with the "canard" system and we further suggest similar analysis of discrete quadratic double centers. Finally, we revisit the 3-dimensional discrete Euler Top.

Title: $\mathbb{S}^1$-actions, semitoric systems, and hyperbolic singularities

Sonja Hohloch
University of Antwerp, Belgium

Abstract:
In this talk, we will discuss recent progress towards a symplectic classification of 2-degree of freedom integrable Hamiltonian systems with $\mathbb{S}^1$-symmetry and mildly degenerate singularities. We give an overview over existing symplectic classifications of toric and semitoric systems and explain the impact of an underlying global $\mathbb{S}^1$-action on a symplectic classification. In particular, we will sketch how to extend an arbitrary effective Hamiltonian $\mathbb{S}^1$-action on a compact 4-dimensional manifold to a 2-degree of freedom integrable systems with only mild degeneracies (if any).

Title: Ray-Singer torsion and the Rumin Laplacian on lens spaces

Akira Kitaoka
University of Tokyo, Japan

Abstract:
The Rumin is the Bernstein-Gelfand-Gelfand complex (BGG complex) of the twisted de Rham complex of a flat vector bundle with respect to contact manifolds. As a typical theorem, the cohomology of the BGG complex coincides with the cohomology of the de Rham complex of a flat vector bundle. Moreover, the Rumin complex arises when we take the sub-Riemannian limit.
Let us consider what happens when we replace a concept defined using the de Rham complex with the Rumin complex. In this talk, we adapt this idea to analytic torsion. On flat vector bundles with a unimodular holonomy over lens spaces, we express explicitly the analytic torsion functions associated with the Rumin complex in terms of the Hurwitz zeta function. In particular, we determine the analytic torsions, and it is written using the Betti numbers and the Ray-Singer torsion.

Title: Some recent results on nonintegrability of dynamical systems

Kazuyuki Yagasaki
Kyoto University, Japan

Abstract:
The problem of nonintegrability is very classical and important in the field of dynamical systems. Its history dates back to the time of Bruns and Poincare in the nineteenth century. In this talk I review my recent results on this topic. First, I describe an outline of the Morales-Ramis theory and illustrate the theory for the SEIR epidemic model as an example. Secondly, I explain a theory developed recently for nonintegrability of nearly integrable systems, and briefly describe its application to the restricted three-body problem and Duffing oscillator. The theory is based on the Morales-Ramis theory. Thirdly, I state my very recent result on the nonintegrability of general three- and four-dimensional systems near degenerate equilibria: The Jacobian matrices of the vector fields have a zero and a pair of purely imaginary eigenvalues for the former systems, and have two pairs of purely imaginary eigenvalues without resonance for the latter ones. Finally, I give some comments on future work.

Title: Teichmüller space and differential character valued momentum maps

Tudor S. Ratiu
Shanghai Jiao Tong University, China / École Polytechnique Fédérale de Lausanne, Switzerland

Abstract:
TBA

Title: Trivializable subriemannian structures on spheres and subelliptic heat equation

Wolfram Bauer
Leibniz University Hannover, Germany

Abstract:
Subriemannian geometry provides a mathematical framework for the study of motions under non-holonomic constraints. Generalizing the Beltrami-Laplace operator in Riemannian geometry a subriemannian structure induces in many examples geometrically defined second order subelliptic differential operators such as the sublaplacian. In recent years, the study of such operators and their corresponding heat equation has reveals interesting links between analysis and geometry. In this talk we consider a specific class of subriemannian structures on Euclidean spheres in [1] which we call trivializable. In particular, on the seven dimensional sphere $\mathbb{S}^7$ we compare a rank 4 trivializable subriemannian structure and a more standard structure induced by the quaterionic Hopf fibration under different geometric and analytic aspects. We consider the corresponding intrinsic sublaplacians and their heat kernel. Explizit formulas and the small time heat kernel asymptotic will be discussed. By choosing appropriate coordinates we can derive the heat kernel of a rank five trivializable structure on $\mathbb{S}^7$ which is then applied to spectral theory and leads to the inverse of the conformal sublaplacian. This talk is based on joint work with Abdellah Laaroussi (Leibniz University Hannover) and Daisuke Tarama (Ritsumeikan University).

[1] W. Bauer, K. Furutani, C. Iwasaki, *Trivializable sub-Riemannian structures on spheres,* Bull. Sci. Math. 137 (2013), no. 3, 361-385.

[2] W. Bauer, A. Laaroussi, *Trivializable and quaternionic subriemannian structure on S^7 and subelliptic heat kernel*, available on `arXiv:2102.04784`, 2021.

Title: Dirac structures over the space of connections on 3 and 4 dimensional Riemannian manifolds

Yuji Hirota
Azabu University, Japan

Abstract:
The study of the space of connections over manifolds, which arose from gauge theory, has been developed involving topology and symplectic geometry. As is well known, the space of connections over Riemannian surfaces was shown to be a symplectic manifold of infinite dimension by M. Atiyah and R. Bott. Afterwards, the space of irreducible connections over four dimensional manifolds with boundary was discussed by T. Kori, and was shown to be a presymplectic manifold. Like these, the space of connections provides us with examples of geometric structures in infinite dimensional cases.
In this talk, we focus on the space of irreducible connections over three and four dimensional manifolds and show that it has a certain geometric structure, so called a Dirac structure. the notion of Dirac structure was first introduced by T. Courant and A. Weinstein, which gives the unified framework of presymplectic geometry and Poisson geometry. We describe explicitly some Dirac structures by using bundle homomorphisms. This is a joint work with Emeritus Professor Tosiaki Kori, Waseda University.

Title:Surjectivity of convolution operators

Tomoyuki Kakehi
University of Tsukuba, Japan

Abstract:
If μ is a compactly supported distribution on $\mathbb{R}^n$, the convolution operator C_μ: $f \mapsto f * \mu$ is well defined as a map from $C^\infty(\mathbb{R}^n)$ to itself. Then we arrive at the following question. When is C_μ surjective? Ehrenpreis gave an answer to this question by showing that C_μ is surjective if and only if the Fourier-Laplace transform $\widehat{\mu}(\lambda)$ of μ is slowly decreasing. In this talk, we start with the above result by Ehrenpreis, and next, we explain that an analogous result holds for convolution operators on noncompact symmetric spaces.

Title: Degenerate Elliptic Boundary Value Problems with Non-smooth Coefficients

Elmar Schrohe
Leibniz University Hannover, Germany

Abstract:
On a manifold X with boundary and bounded geometry we consider a uniformly strongly elliptic second order operator A that locally is of the form

$$A = -\sum_{j,k} a_{jk}\partial_{x_j}\partial_{x_k} + \sum_j b_j\partial_{x_j} + c,$$

together with a boundary operator

$$T = \varphi_0\gamma_0 + \varphi_1\gamma_1,$$

where γ_0 and γ_1 denote the evaluation of a function and its exterior normal derivative, respectively, at the boundary, and φ_0, φ_1 are non-negative C_b^∞ functions on the boundary with $\varphi_0 + \varphi_1 \geq c_0 > 0$. This problem is not elliptic in the sense of Lopatinskij and Shapiro, unless either $\varphi_1 \neq 0$ everywhere or $\varphi_1 = 0$ everywhere.
The condition that φ_0 and φ_1 are smooth can be relaxed to $\varphi_0 = 1$ and $\varphi_1 = \psi^2$ for a function $\psi \in C^{2+\epsilon}$ for some $\epsilon > 0$.
Under the assumption that the a_{jk} are Hölder continuous and b_j as well as c are L^∞ we show

- unique solvability for the boundary value problem $(A - \lambda)u = f, Tu = g$ for $f \in L_p(\Omega)$ and g in a suitable trace space for all sufficiently large $\lambda \notin \mathbb{R}_{>0}$
- the existence of a bounded H_∞-calculus of arbitrarily small angle for the realization A_T of A in $L^p(\Omega)$
- short time solvability for the porous medium equation with the boundary condition.

Moreover, we sketch how these results possibly can be applied to a quasi-stationary Stefan problem for the melting of ice against an inhomogeneous background.
(Joint work with Thorben Krietenstein, Hannover)

Title: Heat kernel asymptotics for quaternionic contact manifolds

Abdellah Laaroussi
Leibniz University Hannover, Germany

Abstract:
We consider the small time asymptotics for the heat kernel associated to the intrinsic sublaplacian on a quaternionic contact manifold considered as a subriemannian manifold. More precisely, we explicitly compute the first two coefficients c_0 and c_1 appearing in the small time asymptotics expansion of the heat kernel on the diagonal. We show that the second coefficient c_1 equals the scalar curvature κ (up to a constant multiple) associated to the canonical connection defined on such a manifold.

Title: Obstructions to integrability of nearly integrable dynamical systems near regular level sets

Shoya Motonaga
Kyoto University, Japan

Abstract:
We consider analytical, nearly integrable systems which may be non-Hamiltonian, and discuss their nonintegrability in the non-Hamiltonian sense. We give a sufficient condition for them to be analytically nonintegrable such that the commutative vector fields and first integrals depend on the perturbation parameter analytically. We compare our results with classical results of Poincar and Kozlov for systems written in action and angle coordinates and discuss their relationships with the subharmonic and homoclinic Melnikov methods for periodic perturbations of single-degree-of-freedom Hamiltonian systems. This is joint work with Kazuyuki Yagasaki (Kyoto University).

Title: Microlocal analysis of d-plane transform on the Euclidean space

Hiroyuki Chihara
University of the Ryukyus, Japan

Abstract:
The d-plane transform is defined by integrals of functions on the n-dimensional Euclidean space over all the d-dimensional planes. where $0 < d < n$. This maps functions on the Euclidean space to those on the affine Grassmannian $G(d, n)$. This is said to be X-ray transform if $d = 1$ and Radon transform if $d = n - 1$. When $n = 2$ the X-ray transform is thought to be measurements of CT scanners. In this talk we begin with the basic properties of the d-plane transform, and talk about concrete expression of the canonical relation of the d-plane transform and quantitative properties of the product of the image of the d-plane transforms. The latter one is related to the streaking artifact of CT image, and some generalization of recent results of Park-Choi-Seo (2017) and Palacios-Uhlmann-Wang (2018) for the X-ray transform on the plane.

Title: Automorphisms of Boutet de Monvel algebra
Ryszard Nest
University of Copenhagen, Denmark

Abstract:
In a remarkable work, Duistermaat and Singer in 1976 studied the algebras of all classical pseudodifferential operators on smooth (boundaryless) manifolds. They gave a description of order preserving algebra isomorphism between the algebras of classical pseudodifferential operators of two manifolds under a cohomological assumption pertaining the first manifold. 'Order preserving' here means that the isomorphism preserves the order of the operators. Surprisingly no continuity assumption is necessary; continuity is automatic. In this talk we will describe constructions involved in generalising their results to the Boutet de Monvel algebra of operators on manifolds with boundary. The main fact of life about manifold with boundary is that vector fields do not define global flows and the "boundary conditions" are a way of dealing with this problem. The Boutet de Monvel algebra corresponds to the choice of local boundary conditions and is, effectively, a non-commutative completion of the manifold. One can think of it as a parametrised version of the classical Toeplitz algebra as a completion of the half-space. Once this is explained, the analysis that we need reduces to a high degree to relatively classical results about automorphisms and homology of the Toeplitz algebra and we will explain what those are and how they are used. This is work in progress joint with Elmar Schrohe.

Title: A unified approach to extremal curves on Stiefel manifolds
Irina Markina
University of Bergen, Norway

Abstract:
We present a unified framework for studying extremal curves on real Stiefel manifolds. We start with a smooth one-parameter family of pseudo-Riemannian metrics on a product of orthogonal groups acting transitively on Stiefel manifolds. We find Euler-Langrange equations for a class of extremal curves that includes geodesics with respect to different Riemannian metrics and smooth curves of constant geodesic curvature. For some specific values of the parameter in the family of pseudo-Riemannian metrics, we recover certain well-known used in applied mathematics. This is a joint work with K. Hueper (University of Wurzburg) and F. Silva Leite (University of Coimbra).

Title: An elliptic fibration arising from the Lagrange top

Genki Ishikawa
Ritsumeikan University, Japan

Abstract:
The Lagrange top is a typical example of integrable systems which describes the motion of a rigid body about a fixed point with symmetric axis through the centre of mass and the fixed point. The complexification of the energy-momentum mapping induces an elliptic fibration over $\mathbb{CP}^2$ in the Weierstraß normal form. One investigates complex algebro-geometric aspects of this elliptic fibration. In particular, the discriminant locus of this elliptic fibration is described in detail and the types of singular fibres are determined according to the classification of singular fibres for elliptic threefolds by R. Miranda.

Title: WKB and microlocal approach to various Bohr-Sommerfeld quantization rules

Setsuro Fujiié
Ritsumeikan University, Japan

Abstract:
The well-known Bohr-Sommerfeld quantization rule is the condition for an energy λ to be an eigenvalue of a 1D Schrödinger operator $P = -h^2\Delta + V(x)$ with a simple well potential $V(x)$. It is an equation of the form

$$-e^{iS(\lambda)/h} = 1,$$

where $S(\lambda) = \int_{\gamma(\lambda)} \xi dx$ is the action along the periodic classical trajectory $\gamma(\lambda)$ in the phase space $T^*\mathbb{R} \sim \mathbb{R}_x \times \mathbb{R}_\xi$ associated with the simple well. A root of this equation gives an approximation of an eigenvalue when the semiclassical parameter h is small.
This fact has been justified rigorously in various mathematical methods such Airy function expansion or exact (complex) WKB method etc. In this talk, we propose another method based on the microlocal analysis. The advantages of this method are the followings:

- We need to construct solutions only along the periodic curve $\gamma(\lambda)$ in the phase space.
- We do not need to care about the divergence of the microlocal WKB expansion.
- This approach is in line of the intuitive interpretation of the rule that the quantum wave should coincide after a tour around $\gamma(\lambda)$ with the original wave.
- This method is naturally adapted to the quantization of quantum resonances in multi-dimension or for systems.

In this talk, this microlocal method is explained with emphasize on the above features and some recent applications are presented.

(3) Organizers

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Hiroaki Yoshimura
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