RIMS 共同研究（公開型）

幾何構造と微分方程式

— 対称性・特異点・力学系の視点から —

Geometric Structures and Differential Equations – Symmetry, Singularity, and Dynamical Systems –

Program and Abstracts

17 – 20 December 2024

Kyoto University
Research Institute for Mathematical Sciences
Room 111

Organizers:
Daisuke Tarama (多羅間 大輔, Ritsumeikan University)
Kenro Furutani (古谷 賢朗, Osaka Metropolitan University / Tokyo University of Science)
Hiroaki Yoshimura (吉村 浩明, Waseda University)

Time Schedule

December 17 (Tuesday)	
13:20 – 13:30	Opening
13:30 – 14:20	01 Elmar Schrohe
14:40 – 15:30	02 Takaharu Yaguchi
15:40 – 16:30	03 Linyu Peng
December 18 (Wednesday)	
9:45 – 10:35	04 Goo Ishikawa
11:00 – 11:50	05 Irina Markina
13:45 – 14:35	06 Sonja Hohloch
14:50 – 15:40	07 Jérémie Pierard de Maujouy
16:00 – 16:50	08 Veronique Fischer
December 19 (Thursday)	
9:45 – 10:35	09 Yoshio Uwano
11:00 – 11:50	10 Chisato Iwasaki
13:30 – 14:20	11 Junya Takahashi
14:40 – 15:30	12 François Gay-Balmaz
15:40 – 16:30	13 Kenro Furutani
December 20 (Friday)	
9:45 – 10:35	14 Genki Ishikawa
11:00 – 11:50	15 Tudor S. Ratiu
11:50 – 12:00	Closing

(0) Basic Information

Dates: From 17/December/2024 till 20/December/2024

Venue: Research Institute for Mathematical Sciences, Kyoto University , Room 111
Participation through Zoom meeting.

Please register through the following URL:
`https://us02web.zoom.us/meeting/register/tZAoceGqrD4sEtZSBRmPzzYWiTbVjx6SsGtu`

Web-site:
`http://www.math.ritsumei.ac.jp/~dtarama/GSDE2024/index.html`

(1) Program

Tuesday, 17/December/2024

13:20 – 13:30 Opening

13:30 – 14:20 Elmar Schrohe (Leibniz University Hannover)
The plasmonic eigenvalue problem, the Calderón projector and the Dirichlet-to-Neumann operator on manifolds with fibered cusp singularities

14:40 – 15:30 Takaharu Yaguchi (Kobe University)
On a posteriori estimates of physics-informed neural networks for solving partial differential equations

15:40 – 16:30 Linyu Peng (Keio University)
The construction of invariant variational integrators via moving frames

Wednesday, 18/December/2024

9:45 – 10:35 Goo Ishikawa (Hokkaido University)
Prolongations of distributions from a viewpoint of sub-Riemannian geometry

11:00 – 11:50 Irina Markina (University of Bergen)
Local invariants and geometry of the sub-Laplacian on H-type foliations

13:45 – 14:35 Sonja Hohloch (University of Antwerp)
On the affine invariant of hypersemitoric systems

14:50 – 15:40 Jérémie Pierard de Maujouy (University of Tours)
The statistical manifold approach to thermodynamics

16:00 – 16:50 Veronique Fischer (University of Bath)
High frequency analysis in sub-Riemannian geometry (online)

Thursday, 19/December/2024

9:45 – 10:35 Yoshio Uwano (Kyoto Pharmaceutical University)
The phase space description of the geodesics on the quantum statistical manifold

11:00 – 11:50 Chisato Iwasaki (University of Hyogo)
Symbolic calculus of pseudo-differential operators on manifolds with conical singularities

13:30 – 14:20 Junya Takahashi (Tohoku University)
Comparison of the eigenvalues of the Hodge-Laplacian and special holonomy groups

14:40 – 15:30 François Gay-Balmaz (Nanyang Technological University)
Geometry and numerics of Navier-Stokes-Fourier fluids (online)

15:40 – 16:30 Kenro Furutani (Osaka Metropolitan University / Tokyo University of Sciences)
Lagrangian submanifold satisfying Maslov's quantization condition

Friday, 20/December/2024

9:45 – 10:35 Genki Ishikawa (Ritsumeikan University)
Stability analysis for the pseudo-Riemannian geodesic flows of step-two nilpotent Lie groups

11:00 – 11:50 Tudor S. Ratiu (Shanghai Jiao Tong University)
The Teichmüller space obtained by reduction

11:50 – 12:00 Closing

(2) Abstracts

Title: The Plasmonic Eigenvalue Problem, the Calderón Projector and the Dirichlet-to-Neumann Operator on Manifolds with Fibered Cusp Singularities

Elmar Schrohe
Leibniz University Hannover, Germany

Abstract:
A plasmon of a bounded domain $\Omega \subseteq \mathbb{R}^n$ is a nontrivial harmonic function on $\mathbb{R}^n \setminus \partial\Omega$ which is continuous at $\partial\Omega$ and whose interior and exterior normal derivative at $\partial\Omega$ have a constant ratio. This ratio is called a plasmonic eigenvalue of Ω. It is indeed an eigenvalue of $N_+^{-1}N_-$, where $N_\pm$ denote the exterior and interior Dirichlet-to-Neumann operators.
Our longterm term goal is to understand this problem on a manifold with fibered cusp singularities. A prototypical example would be the complement of two touching strictly convex domains in $\mathbb{R}^n$. Clearly, the problem requires a precise analysis of the Dirichlet-to Neumann operator in this setting. In a first step, we consider the Calderòn projector for general elliptic differential operators of arbitrary order associated with this type of singularity, so-called ϕ-differential operators. We show that the Calderòn projector is a ϕ-pseudodifferential operator in the sense of Mazzeo and Melrose. Next we study the Dirichlet-to-Neumann operator for Laplacians associated with fibered cusp metrics and obtain that it also is a ϕ-pseudodifferential operator of order one.
This is a report on ongoing work with Karsten Fritzsch and Daniel Grieser.

Title: On a posteriori estimates of physics-informed neural networks for solving partial differential equations

Takaharu Yaguchi
Kobe University, Japan

Abstract:
Physics-Informed Neural Networks (PINNs) are applications of neural networks to initial-boundary-value problems of partial differential equations. Specifically, PINNs can be used to solve the partial differential equations numerically and/or to estimate parameters of the equations from given observational data. In this talk, we will give an error estimate for PINNs for solving partial differential equations in a different way from existing analysis.

Title: The construction of invariant variational integrators via moving frames

Linyu Peng
Keio University, Japan

Abstract:
In this talk, we introduce a novel method for constructing invariant variational integrators by incorporating the framework of moving frames. Variational integrators are well known for their ability to preserve the symplectic structure, making them highly effective for long-term simulations. However, conventional variational integrators may fail to preserve the symmetries and corresponding conserved quantities of a system. By utilizing moving frames, we develop integrators that not only maintain the symplectic nature of the system but also preserve the system's intrinsic symmetries and invariants. We will discuss the mathematical foundation of this construction and demonstrate its advantages through various mechanical models. This talk is based on joint works with Mamoru Gunji, Peter Hydon, Elizabeth Mansfield, and Ana Rojo-Echeburúa.

Title: Prolongations of distributions from a viewpoint of sub-Riemannian geometry

Goo Ishikawa
Hokkaido University, Japan

Abstract:
A sub-Riemannian structure on a manifold is given by a vector sub-bundle of the tangent bundle (=distribution) endowed with a Riemannian (symmetric, positive definite) metric on the sub-bundle. Then, in sub-Riemannian geometry, there arise naturally the notions of "abnormal geodesics" and "singular curves" in addition to normal geodesics. Singular curves are defined as singularities of "end-point mapping", which is independent of the metric of the sub-bundle, and the appearance of singular curves is regarded as an invariant character of the distribution. In this talk, we explain a way to prolong distributions of growth types $(2, 3, 5)$, $(4, 7)$ and $(8, 15)$, and to obtain graded Lie algebras of types G_2, C_3 and F_4, using data on their singular curves.

Title: Local invariants and geometry of the sub-Laplacian on H-type foliations

Irina Markina
University of Bergen, Norway

Abstract:
Let (M, g) be a smooth, oriented, connected Riemannian manifold equipped with a Riemannian foliation with bundle-like complete metric g and totally geodesic leaves satisfying some additional symmetry conditions. The manifold is studied in the framework of sub-Riemannian geometry with bracket generating distribution transversal to the totally geodesic fibers. Equipping M with the Bott connection we find local invariants by studying the small-time asymptotics of the sub-Riemannian heat kernel. We obtain the first three terms in the asymptotic expansion of the Popp volume for the pull-back of small sub-Riemannian balls. We address also the question of local isometry of M as a sub-Riemannian manifold and its tangent group.
This is the joint work with W. Bauer, A. Laaroussi (Leibnitz University of Hannover, Germany), S. Vega-Molino (Naval Academy of the Norwegian Defense Academy)

References

[1] W. Bauer, I. Markina, A. Laaroussi, G. Vega-Molino, Local Invariants and Geometry of the sub-Laplacian on H-type Foliations. arXiv preprint 2209.02168
[2] F. Baudoin, E. Grong, L. Rizzi, G. Vega-Molino, H-type Foliations. Differential Geom. Appl. 85 (2022), Paper No. 101952, 25 pp.

Title: On the affine invariant of hypersemitoric systems

Sonja Hohloch
University of Antwerp, Belgium

Abstract:
In 1988, Delzant symplectically classified toric integrable systems by means of their momentum map image which is a very nice and special convex polytope, often referred to as 'Delzant polytope' of the toric system.
Semitoric systems are integrable systems of the form $F = (J, H) : (M, \omega) \to \mathbb{R}$ where (M, ω) is a 4-dimensional connected symplectic manifold and J is proper and induces an effective Hamiltonian torus action and F admits only nondegenerate singularities and no hyperbolic components. Intuitively, semitoric systems generalize toric systems in dimension four by admitting in addition to elliptic-elliptic and elliptic-regular singularities also focus-focus singularities. In 2009-2011, Pelayo & Vu Ngoc symplecticaly classified semitoric integrable systems in terms of 5 invariants, among which a 'generalized semitoric polytope' deduced from the momentum map image, i.e. generalizing the Delzant polytope.
When admitting also hyperbolic components for the nondegenerate singularities and mildly degenerate (so-called parabolic) points, then one generalizes semitoric systems to so-called hypersemitoric systems. The longterm goal is to obtain a classification of hypersemitoric integrable systems on compact connected 4-dimensional symplectic manifolds.
This talk presents one of the expected invariants, the so-called 'affine invariant' which is the generalization of the semitoric polytope invariant. This talk is based on ongoing work with N. Flamand (Antwerp) and a joint preprint (arXiv:2411.17509) with K. Efstathiou (Duke Kunshan University) and P. Santos (Antwerp).

Title: The statistical manifold approach to thermodynamics

Jérémie PIerard de Maujouy
University of Tours, France

Abstract:
Although the laws of thermodynamic appear very different from the law of mechanics, the behaviour of a thermodynamic system is in principle determined by the behaviour of its microscopic components, which follow the laws of mechanics. This approach to thermodynamics is called statistical mechanics.
In geometric approaches, a mechanical system with symmetry is modelled by a Hamiltonian G-manifold and a "statistical state" is a probability distribution on the manifold. In Structure of Dynamical Systems, Souriau associates to a mechanical system a differentiable manifold that describes the states of thermodynamic equilibrium of the corresponding statistical mechanical system. The equilibrium states form what is called in modern language a statistical transformation model: a finite-dimensional family of probability distributions with an action of the symmetry group.
In this construction, the temperature acquires a geometrical character and appears as a component of a vector called "generalised temperature" which lives in the Lie algebra of infinitesimal symmetries. We will introduce this little-known construction and discuss coadjoint orbits as well as the physical system of a rotating ideal gas.

Title: High frequency analysis in sub-Riemannian geometry

Veronique Fischer
University of Bath, United Kingdom

Abstract:
The aim of the talk is to present recent developments of high frequency analysis for subelliptic operators and in sub-Riemannian geometry. I will start with discussing why these questions are closely related to many aspects of harmonic analysis.

The phase space description of the geodesics on the quantum statistical manifold

Yoshio Uwano
Kyoto Pharmaceutical University, Japan

Abstract:
The geodesics on the statistical manifolds have drawn attentions in both classical and quantum information geometry. As characteristic geodesics, the mixture (m-)geodesics, the Riemannian (R-) ones, and the exponential (e-) ones are known very well. In this talk, these three kinds geodesics are studied on the quantum statistical manifold, the space of density matrices endowed with the Fisher metric, from dynamical systems viewpoint. The cotangent bundle of the quantum statistical manifold is taken as the phase space on which the equations for the three kinds of geodesics are given. The equations thus obtained are solved and studied from the integrability and the Hamiltonian mechanical viewpoints. A review of the classical statistical case is given for comparison.

Symbolic calculus of pseudo-differential operators on manifolds with conical singularities

Chisato Iwasaki
University of Hyogo, Japan

Abstract:
TBA

Comparison of the eigenvalues of the Hodge-Laplacian and special holonomy groups

Junya Takahashi
Tohoku University, Japan

Abstract:
For the eigenvalues of the Hodge Laplacian acting on differential forms on an oriented closed Riemannian manifold, we explore how the first positive eigenvalue of the Hodge-Laplacian acting on p-forms depends on the degree p.
We study this problem, when a closed Riemannian manifold admits a non-trivial parallel q-form, whose existence implies a special holonomy group. This work generalizes one of Arapura in 2018 dealing with Kähler manifolds to the other special holonomy manifolds: quaternionic Kähler manifolds, G_2-manifolds and Spin(7)-manifolds.
This is a joint work with Colette Anné (Nantes Univ. in France).

Geometry and Numerics of Navier-Stokes-Fourier fluids

François Gay-Balmaz
Nanyang Technological University, Singapore

Abstract:
We construct a structure-preserving and thermodynamically consistent finite element method and time stepping scheme for heat conducting viscous fluids. The method is derived by discretizing a variational formulation for nonequilibrium thermodynamics that extends Hamilton's principle for fluids to systems with irreversible processes. The resulting scheme preserves energy and mass balance to machine precision, while also ensuring compliance with the second law of thermodynamics, both at the spatially and temporally discrete levels. The method is shown to apply both with insulated and prescribed heat flux boundary conditions, as well as with prescribed temperature boundary conditions. The effectiveness of the scheme is demonstrated through its application to Rayleigh-Bénard convection. Some current direction of research will be given.

Lagrangian submanifold satisfying Maslov's quantization condition

Kenro Furutani
Osaka Metropolitan University, Japan

Abstract:
In this talk I will explain so called Maslov's quantization condition and a related example. Starting from Bohr's hydrogen model, a classical but famous condition Maslov's quantization condition was an interesting subject, which guarantees the existence of a certain sequence of eigenvalues of the Laplacian. The corresponding eigenfunctions will be eigenstates in quantum mechanical phenomena. So after some introduction, I explain an operator theoretical meaning of the role deduced by the existence of Lagrangian submanifolds, particularly its meaning from the Fourier integral operator theory together with a brief introduction of Maslov index and Maslov class.
The main contents of this talk is to show such a Lagrangian submanifold on the Cayley projective plane.
The construction is explicit based on the realization of the punctured cotangent bundle of the Cayley projective in the complex space $\mathbb{C}^{27}\backslash\{0\}$ as a quadric.
It is not clear within my understanding that the existence of such Lagrangian submanifold on any Riemannian manifold, especially in the case of the geodesic flow being ergodic.

Title: Stability analysis for the pseudo-Riemannian geodesic flows of step-two nilpotent Lie groups

Genki Ishikawa
Ritsumeikan University, Japan

Abstract:
This talk deals with the geodesic flows of step-two nilpotent Lie groups equipped with a left-invariant pseudo-Riemannian metric. The left-invariant geodesic flow of a Lie group can be formulated as the Lie-Poisson equation on the dual space of its Lie algebra. In particular, in the case of step-two nilpotent Lie groups, the Lie-Poisson equation can be described in terms of the so-called j-mapping, a linear operator associated to the step-two nilpotent Lie algebras equipped with the induced scalar product. In this talk, the stability of equilibrium points for the Hamilton equation is discussed in view of their Williamson types. This talk is based on a joint work with Daisuke Tarama (Ritsumeikan Univ.).

Title: Teichmüller space and differential character valued momentum maps

Tudor S. Ratiu
Shanghai Jiao Tong University, China / École Polytechnique Fédérale de Lausanne, Switzerland

Abstract:
Two different ways to obtain the Teichmüller space as a reduced space are presented. The first one uses the standard momentum map in a non-equivariant context. The second one uses a differential character valued momentum map, whose definition and properties will be also explained.

(3) Organizers

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