RIMS 共同研究（公開型）

# 幾何構造と微分方程式

## — 対称性・特異性・応用の視点から —

# Geometric Structures and Differential Equations
## – Symmetry, Singularity, and Applications –

# Program and Abstracts

**13 – 17 July 2026**

**Kyoto University**
**Research Institute for Mathematical Sciences**
**Room 111**

**Organizers:**
Kenro Furutani (古谷 賢朗, Osaka Metropolitan University / Tokyo University of Science)
Shoya Motonaga (本永 翔也, Kyushu Institute of Technology)
Daisuke Tarama (多羅間 大輔, Ritsumeikan University)
Hiroaki Yoshimura (吉村 浩明, Waseda University)

## Time Schedule

| | |
|---|---|
| July 13 (Monday) | |
| 12:50 – 13:00 | Opening |
| 13:00 – 14:00 | 01 Kazuyuki Yagasaki |
| 14:15 – 15:15 | 02 Keiichi Sakai |
| 15:30 – 16:30 | 03 Irina Markina |
| July 14 (Tuesday) | |
| 10:00 – 11:00 | 04 Holger Dullin |
| 11:15 – 12:15 | 05 Yusaku Tiba |
| 14:00 – 15:00 | 06 Shouhei Honda |
| 15:15 – 16:15 | 07 Yoshihiro Fukumoto |
| July 15 (Wednesday) | |
| 10:00 – 11:00 | 08 Chisato Iwasaki |
| 11:15 – 12:15 | 09 Takaharu Yaguchi |
| 14:00 – 15:00 | 10 Ursula Ludwig |
| 15:15 – 16:15 | 11 Yurii Nikonorov |
| July 16 (Thursday) | |
| 10:00 – 11:00 | 12 Junya Takahashi |
| 11:15 – 12:15 | 13 Hajime Fujita |
| 14:00 – 15:00 | 14 Jérémie Pierard de Maujouy |
| 15:15 – 16:15 | 15 Kiyoshi Takeuchi |
| July 17 (Friday) | |
| 11:00 – 12:00 | 16 Shigeki Matsutani |
| 13:30 – 14:30 | 17 Konstantinos Efstathiou |
| 14:40 – 15:40 | 18 Toshihiro Iwai |
| 15:40 – 15:50 | Closing |

# (0) Basic Information

Dates: From 13/July/2026 till 17/July/2026

Venue: Research Institute for Mathematical Sciences, Kyoto University , Room 111
Participation through Zoom meeting.

Please register through the following URL:
`https://us06web.zoom.us/meeting/register/D3GOjrYHQtSCX9ZVehotIg`

Web-site:
`http://www.math.ritsumei.ac.jp/~dtarama/GSDE2026/index.html`

# (1) Program

Monday, 13/July/2026

12:50 – 13:00 Opening

13:00 – 14:00 Kazuyuki Yagasaki (Kyoto University)
*Linearization and integrability of continuous and discrete dynamical systems*

14:15 – 15:15 Keiichi Sakai (Shinshu University)
*Diffeological spaces and their Riemannian structures*

15:30 – 16:30 Irina Markina (University of Bergen) online
*Nilpotent Lie groups and ultra-hyperbolic differential operators*

Tuesday, 14/July/2026

10:00 – 11:00 Holger Dullin (University of Sydney)
*The $n$-body problem on coadjoint orbits*

11:15 – 12:15 Yusaku Tiba (Ochanomizu University)
*On the Bohr–Sommerfeld condition and the convergence of holomorphic sections*

14:00 – 15:00 Shouhei Honda (University of Tokyo)
*Gel'fand's inverse problem under Ricci curvature bounds*

15:15 – 16:15 Yoshihiro Fukumoto (Ritsumeikan University)
*Involutions on 4-dimensional orbifold cobordisms*

Wednesday, 15/July/2026

10:00 – 11:00 Chisato Iwasaki (University of Hyogo)
*Heat kernels for Laplacians on infinite cones in low dimensions*

11:15 – 12:15 Takaharu Yaguchi (Kyushu University)
*Methods for learning symplectic forms*

14:00 – 15:00 Ursula Ludwig (University of Côte d'Azur)
*Analytic torsion on singular spaces*

15:15 – 16:15 Yurii Nikonorov (Southern Mathematical Institute of VSC RAS) online
*Properties of plane angles of tetrahedra with a given base*

## Thursday, 16/July/2026

10:00 – 11:00 Junya Takahashi (Tohoku University)
*Comparison of the eigenvalues of the Hodge-Laplacian and special differential forms*

11:15 – 12:15 Hajime Fujita (Japan Women's University)
*On a compactification of the orthogonal foliation via toric geometry*

14:00 – 15:00 Jérémie Pierard de Maujouy
*Statistical cones from thermodynamics on nilpotent coadjoint orbits*

15:15 – 16:15 Kiyoshi Takeuchi (Tohoku University)
*Lagrangian cycles and constructible sheaves*

## Friday, 17/July/2026

11:00 – 12:00 Shigeki Matsutani (Kanazawa University)
*Algebro-geometric solutions of the MKdV equation and the supercoiled structure of DNA*

13:30 – 14:30 Konstantinos Efstathiou (Duke Kunshan University)
*Singular Lagrangian fibration of a Tavis-Cummings system with an* $A_2$ *singularity*

14:40 – 15:40 Toshihiro Iwai (Kyoto University)
*Jahn-Teller systems with cubic symmetry in the bundle formulation*

15:40 – 15:50 Closing

## (2) Abstracts

**Title: Linearization and integrability of continuous and discrete dynamical systems**

Kazuyuki Yagasaki
Kyoto University, Japan

**Abstract:** Linearization and integrability of dynamical systems, typically given by differential equations in continuous time and by diffeomorphisms in discrete time, are among the most important topics in the theory of dynamical systems.
In this talk, I introduce my recent results on the linearization and integrability of continuous and discrete dynamical systems. The main tools used to obtain these results are the Koopman operator in the discrete case and the Lie operator, the infinitesimal generator of the Koopman operator group, in the continuous case. In particular, I formulate a new definition of integrability for general diffeomorphisms, motivated by Bogoyavlenskij's definition of integrability for general differential equations. I illustrate the theory through classical and representative examples: the Duffing oscillator, the van der Pol oscillator, the Lorenz system, the restricted three-body problem, the Henon map, and the standard map.

**Title: Diffeological spaces and their Riemannian structures**

Keiichi Sakai
Shinshu University, Japan

**Abstract:** A diffeology of a set $X$ is a generalization of atlas of smooth manifolds and characterizes which maps from/to $X$ are "smooth". Diffeology was first initiated by J.-M. Souriau in 1980's to extend the category of smooth manifolds to a convenient one. Thus, diffeological spaces (sets equipped with diffeology, called diff-spaces for short) generalize smooth manifolds and include a lot of important examples; spaces with singularities (orbifolds, ...), mapping spaces of manifolds, and so on. Diff-spaces have been mainly studied from the viewpoint of algebraic topology and homotopy theory, and their differential-geometric foundation is still to be established. In this talk, after a quick review of foundations of diff-spaces, we would introduce the notion of weak Riemannian metric on diff-spaces, based on C. Blohmann's "tangent functor". We would see various examples; weak Riemannian manifolds of infinite dimension, adjunction spaces and mapping spaces, and some important map in string topology that turns out to be an isometry. This talk is based on joint work with Katsuhiko Kuribayashi (Shinshu U) and Yusuke Shiobara (Shinshu U).

**Title: Nilpotent Lie groups and ultra-hyperbolic differential operators**
Irina Markina
University of Bergen, Norway

**Abstract:**
In this talk, we introduce a special class of nilpotent Lie groups called Heisenberg-type groups. We will also discuss the difference between elliptic and hyperbolic operators on Euclidean space and their analogues on Lie groups.
Finally, we will define a natural differential operator on Heisenberg type Lie groups. We will discuss the fundamental solution of this operator, as well as related results found in the literature.
This is the joint work with Wolfram Bauer and Andre Froehly, Leibniz University Hannover, Germany.

**Title: The $n$-body problem on coadjoint orbits**
Holger Dullin
University of Sydney, Australia

**Abstract:** Symmetry reduction of the $N$-body problem using invariants of the symmetry group action leads to a description of the dynamics on coadjoint orbits of the symplectic group $Sp(2n-2)$. As a result the symmetry reduced equations can be written as a Lax pair. The coadjoint orbits are classified by two integer invariants and by spectral invariants. The integer invariants are the rank of a Gram matrix containing the invariants and of the angular momentum matrix. The spectral invariants are determined by the spectrum of the angular momentum matrix. As an "application" we show that the 3-body problem in 4-dimensional space has Lyapunov stable relative periodic orbits.

**Title: On the Bohr–Sommerfeld condition and the convergence of holomorphic sections**
Yusaku Tiba
Ochanomizu University, Japan

**Abstract:** We introduce the asymptotic behavior of sequences of holomorphic sections along Bohr–Sommerfeld Lagrangian submanifolds for high tensor powers of a holomorphic prequantum line bundle over a Kähler manifold. The Bohr–Sommerfeld condition arises as the condition for the global construction of WKB solutions on cotangent bundles. In this talk, we consider Bohr–Sommerfeld Lagrangian submanifolds in Kähler manifolds and discuss topics such as convergence to coherent states obtained by holomorphically regularizing constant sections, as well as estimates of sub-mean value inequality type.

**Title: Gel'fand's inverse problem under Ricci curvature bounds**
Shouhei Honda
University of Tokyo, Japan

**Abstract:** The classical Gel'fand's inverse problem asks whether a Riemannian manifold is uniquely determined by the knowledge of the heat kernel on any open subset of the manifold. We study this inverse problem in the non-smooth setting in the framework of $RCD(K, N)$ spaces, namely, metric-measure spaces with synthetic Riemannian Ricci curvature bounded below by $K$ and dimension bounded above by $N$. We establish the unique solvability of Gel'fand's inverse problem for the class of compact $RCD(K, N)$ spaces whose regular set admits $C^1$-Riemannian structure. As an application, we obtain the stability of Gel'fand's inverse problem in the class of closed Riemannian manifolds with bounded Ricci curvature, diameter and volume bounded from below. We note that the results are new even for Einstein orbifolds and (weighted) Riemannian manifolds with non-smooth boundary. This is a joint work with Jinpeng Lu (University of Helsinki), based on arXiv:2602.14527.

**Title: Involutions on 4-dimensional orbifold cobordisms**
Yoshihiro Fukumoto
Ritsumeikan University, Japan

**Abstract:** Applications of gauge theory on 4-orbifolds was initiated by Fintushel and Stern to study homology cobordism group of homology 3-spheres by introducing their R-invariant using instanton moduli spaces in Donaldson theory. In a joint work with Furuta, we introduced a Seiberg-Witten theory counter part of the R-invariant called "$w$-invariant" by using moduli spaces of monopoles on 4-orbifolds. By a compactness of monopole moduli spaces Furuta developed a technique of finite dimensional approximation of the monopole equation to show "10/8-inequality" whose orbifold version led us to prove a homology cobordism invariance of the $w$-invariant for spin orbifolds. Currently, 10/8-inequality is generalized to 4-manifold with boundary by Manolescu by using his kappa invariant from Seiberg-Witten Floer stable homotopy type. Furthermore, if a 4-manifold admits an "odd" involution, Konno-Miyazawa-Taniguchi recently developed a Real version of Seiberg-Witten theory to show a Real 10/8-inequality by using Real kappa invariant and give an obstruction to sliceness of knots for example. In this talk, we describe several constructions of cobordisms by 4-orbifolds with involutions to apply $w$-invariants and discuss relationship with Real kappa invariants. As an application, we give lower bounds of stabilization numbers of connected sums the $(2, 1)$-cable of the figure-eight knot for example. This talk is based on a joint work with Masaki Taniguchi.

**Title: Heat kernels for Laplacians on infinite cones in low dimensions**
Chisato Iwasaki
University of Hyogo, Japan

**Abstract:** TBA

**Title: Methods for learning symplectic forms**
Takaharu Yaguchi
Kyushu University, Japan

**Abstract:** In recent years, research on learning Hamiltonian equations from observational data has attracted significant attention. However, most existing studies are based on Hamiltonian equations in standard form, which are formulated in terms of generalized momenta. Since the analytical expressions for generalized momenta are generally unknown, methods based on the standard form are difficult to apply to real-world data. Neural Symplectic Forms were proposed as a coordinate-free method for learning symplectic structures; however, this method is known to be unstable. In this study, we identify the cause of this instability by investigating theoretical properties of the path connectivity of the set of symplectic forms in the case where the phase space is Euclidean. Specifically, we show that the set of symplectic forms has two path-connected components. Based on this, we propose a deep learning method that can simultaneously explore both connected components.

**Title: Analytic torsion on singular spaces**
Ursula Ludwig
University of Côte d'Azur, France

**Abstract:**
Analytic torsion is an important secondary spectral invariant of compact Riemannian manifolds introduced by Ray and Singer. By the famous Cheeger-Müller theorem the analytic torsion equals the Reidemeister-Franz torsion, hence turns out to be indeed a topological invariant. The higher analytic torsion forms of Bismut and Lott, generalise the concept of analytic torsion to a family of smooth manifolds.
In this talk we will present work in progress on analytic torsion resp. higher analytic torsion for spaces with isolated conical singularities resp. families of such. In the first part of the talk, we will give a gentle introduction of these notions in the classical setting.

**Title: Properties of plane angles of tetrahedra with a given base**

Yurii Nikonorov
Southern Mathematical Institute of the Vladikavkaz Scientific Center of the Russian Academy of Sciences, Russia

**Abstract:**
This talk is based on the joint project with Evgenii V. Niktenko (Rubtsovsk Industrial Institute of Altai State Technical University after I. I. Polzunov, Rubtsovsk, Russia).

Let $\Omega(\triangle ABC)$ be the set of all tetrahedra $ABCD$ in three-dimensional Euclidean space with a given non-degenerate base $ABC$ and a vertex $D$ lying outside the plane $ABC$. Let us consider the set

$$\Sigma(\triangle ABC) = \{(\cos\alpha, \cos\beta, \cos\gamma) \in \mathbb{R}^3 | ABCD \in \Omega(\triangle ABC)\},$$

where $\alpha = \angle BDC$, $\beta = \angle ADC$, and $\gamma = \angle ADB$. The talk is devoted to describing the closure of the set $\Sigma(\triangle ABC)$ in $\mathbb{R}^3$.
The main results were obtained in the following paper:

E.V. Nikitenko, Yu.G. Nikonorov, *On face angles of tetrahedra with a given base.* Results in Mathematics, 81, article number 60 (2026), 50 pp., `https://doi.org/10.1007/s00025-026-02610-x`.

**Title: Comparison of the eigenvalues of the Hodge-Laplacian and special differential forms**

Junya Takahashi
Tohoku University, Japan

**Abstract:** On a closed Riemannian manifold $(M, g)$, we explore how the first eigenvalue $\lambda_1'^{(p)}(M, g)$ of the Hodge-Laplacian acting on exact $p$-forms depends on the degree $p$. If a closed Riemannian manifold admits a special differential form, such as a parallel form and a special Killing form, then we study the relationship between $\lambda_1'^{(p)}(M, g)$ and $\lambda_1'^{(s)}(M, g)$. As a consequence, we obtain an estimate for the gap $\lambda_1'^{(p)}(M, g) - \lambda_1'^{(s)}(M, g)$. This talk is based on joint works with Colette Anné (Nantes Université in France).

**Title: On a compactification of the orthogonal foliation via toric geometry**

Hajime Fujita
Japan Women's University, Japan

**Abstract:** Information geometry provides a unified geometric framework for understanding various constructions in statistics, including applications to machine learning.
One of the key geometric structures in this framework is a Hessian structure, also known as a dually flat structure.
In many important applications, Hessian structures are studied on open subsets of Euclidean spaces.
In particular, the interior of the probability simplex, which parametrizes probability distributions on a finite sample space, naturally carries a Hessian structure.
On the other hand, a Hessian structure is not a priori defined on the boundary of the domain. For example, the boundary of the probability simplex corresponds to probability distributions for which certain events have zero probability, but the Hessian structure does not naturally extend to this boundary.
To address this issue, we propose an approach to compactification using toric geometry. In the case of the probability simplex, we consider a compactification via complex projective space which is a toric manifold whose momentum polytope is the probability simplex.
In this talk, we discuss the orthogonal foliation associated with a Hessian structure and a compactification thereof. The main ingredients are the torus orbit closure, quadratic projective surface and its CR distribution.
This is a joint work with K. Yamaguchi (RIMS).

**Title: Statistical cones from thermodynamics on nilpotent coadjoint orbits**

Jérémie Pierard de Maujouy

**Abstract:** We introduce Souriau's geometric approach to thermodynamics and explain how it associates a statistical manifold to every Hamiltonian G-manifold. In the simple case of coadjoint orbits, these statistical manifolds turn out to be cones. We discuss an example in which the associated statistical manifold is a symmetric space, suggesting a connection with the geometry of symmetric cones. After a brief review of the structure of symmetric cones, we present a family of nilpotent coadjoint orbits for which the associated statistical manifolds are indeed a symmetric cone.
Joint work with P. Bieliavsky and G. Neuttiens.

**Title: Lagrangian cycles and constructible sheaves**

Kiyoshi Takeuchi
Tohoku University, Japan

**Abstract:** Kashiwara's index theorem for constructible sheaves is a vast generalization of the classical Poincaré-Hopf theorem and plays important roles in various branches of mathematics. In his theory, Lagrangian cycles naturally correspond to constructible sheaves (functions). We will report on our recent progress in this correspondence. This is a joint work with Ren Fernandes and Kazuki Kudomi.

**Title: Algebro-geometric solutions of the MKdV equation and the supercoiled structure of DNA**

Shigeki Matsutani
Kanazawa University, Japan

**Abstract:** In addition to its helical structure, DNA has a global supercoiled structure. Although this supercoiledl structure is governed by weak elastic forces, its geometric shape has yet to be described mathematically. Geometric models known as Euler's elastica, which minimise Euer-Bernoulli elastic energy, cannot describe the shape of DNA, even when three-dimensional effects are considered. Since 1997, the speaker has been conducting research into mathematically describing this shape while taking into account the effects of finite temperatures.

Elementary analysis shows that, at finite temperatures, the shape of the curve is described by hyperelliptic function solutions to the modified Korteweg-de Vries (KdV) equation – a nonlinear integrable equation – in two-dimensional space and by nonlinear Schrödinger equation solutions in three-dimensional space. However, as of 1997, the algebraic function theory – including the theory of hyperelliptic functions – had not yet evolved to the point where hyperelliptic function solutions could be described and discussed concretely. Therefore, from 2003 onwards, the speaker and his collaborator, Emma Previato, have been reconstructing the algebraic function theory to bring it up to the level of elliptic function theory, while also developing related theoretical frameworks. As a result, in 2022, they achieved a certain form, albeit an incomplete one. They demonstrated that the real part of the MKdV equation defined over the complex numbers corresponds to the gauge-field-modified MKdV (GMKdV) equation.

Recently the speaker obtained solutions for the GMKdV equation of general genus and, through numerical visualisation, reproduced the shape of DNA observed in the laboratory. This means that, in addition to describing DNA – the origin of life – algebraic geometry can now also represent the excited states of geometric objects with energy functionals. It should be noted that the excited states of real physics, as observed in laboratories, are not solely determined by topological data and this study differ significantly from string theory and related models.

Furthermore, Euler's elastica is the origin of dynamical systems; in fact, following his research on the elastica (1744), Euler discovered the variational method in 1744 and the Newton equations in 1752 (the notion that the Newton equations were derived by Newton is a misconception propagated by the experimentalist Mach). This study on the excited states of elastica must be a prototype for elucidating the structure of excited states in dynamical systems and in geometric objects possessing energy functionals.

This talk presents the results obtained in recent years and explains the processes that led to them.

**Title: Singular Lagrangian fibration of a Tavis-Cummings system with an $A_2$ singularity**

Konstantinos Efstathiou
Duke Kunshan University, China

**Abstract:**
Singular Lagrangian fibrations arising from three-degree-of-freedom integrable Hamiltonian systems remain largely unexplored. While several results describe the global structure of large classes of systems with two degrees of freedom, only a few examples are understood in higher dimensions. In this talk we present a three-degree-of-freedom system derived from the two-spin Tavis-Cummings model whose singular Lagrangian fibration exhibits a topology that has not been observed in other physical models. In particular, the most degenerate singular fiber is homeomorphic to $S^2 \times S^1$ with a singularity of $A_2$ type. We describe the bifurcation diagram and the global topology of the fibration, we compute its Hamiltonian monodromy, and we compare it to the Picard-Lefschetz monodromy of the $A_2$ singularity. Based on joint work with Gabriela Gutierrez-Guillen, Pavao Mardešić, and Dominique Sugny.

**Title: Jahn-Teller systems with cubic symmetry in the bundle formulation**

Toshihiro Iwai
Kyoto University, Japan

**Abstract:** Interest of this talk will center on Jahn-Teller systems with cubic symmetry in the bundle formulation, under the prompting of the paper entitled "Berry phase and the $\Gamma_8 \otimes (\tau_2 \oplus \epsilon)$ Jahn-Teller system" by Aspel-Chancy-O'Brien (Phys. Rev. **B45**, 5251-5261, (1992)). The Jahn-Teller theory is a branch of quantum chemistry for molecules with symmetry by finite subgroups such as $T_d$ and $O$ of $O(3)$, where $T_d$ and $O$ refer to achiral tetrahedral and octahedral symmetries, respectively. However, in place of explaining chemical words such as $\Gamma_8 \otimes (\tau_2 \oplus \epsilon)$, this talk has interest in a mathematical framework of the Jahn-Teller theory. In the vector bundle formulation, the present Jahn-Teller Hamiltonian is defined on a rank-two vector bundle over $\dot{\mathbb{R}}^5 := \mathbb{R}^5 \setminus \{0\}$, and a relevant Berry curvature for the model by ACO'B is shown to be exactly Yang's monopole field (J. Math. Phys. 19, 320-328 (1978)). Furthermore, the formulation is closely related to an old paper of mine on the quantized $SU(2)$ Kepler problem (J. Geom. Phys. **20**, 250-272 (1996)). In the course of solving the eigenvalue problem, the Bochner Laplacian for the rank-two vector bundle over $S^4 \subset \dot{\mathbb{R}}^5$ is exactly solved. From the viewpoint of Jahn-Teller theory, the second Chern number is viewed as an index of a singularity, where a point is viewed as a singularity if at the point the eigenvalues of the electronic Hamiltonian are totally degenerate. Here, the electronic Hamiltonian is defined so as to describe the interaction between electronic states ($\Gamma_8$) and nuclear vibrations ($\tau_2 \oplus \epsilon$) in the Born-Oppenheimer approximation.

## (3) Organizers

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