2018RIMS 共同研究（公開型）

幾何構造と微分方程式

ー 対称性と特異点の視点から ー

Symmetry and Singularity of Geometric Structures and Differential Equations

Program and Abstracts

18 – 21 December 2018

Ritsumeikan University
Biwako-Kusatsu Campus
Westwing 6F Colloquium room

Organizers:
Daisuke Tarama (多羅間 大輔, Ritsumeikan University)
Kenro Furutani (古谷 賢朗, Tokyo University of Science)
Hiroaki Yoshimura (吉村 浩明, Waseda University)

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科研費 KAKENHI

Program Table

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15:00 – 15:20	tea time
15:20 – 16:10	03 Wolfram Bauer
16:20 – 17:10	04 Tohru Morimoto
December 19 (Wednesday)	
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(1) Program

Tuesday, 18/December/2018

13:00 – 13:10 Opening

13:10 – 14:00 Jean-Pierre Françoise
Integrable Systems: The Clebsch top and its associated Kummer Surface

14:10 – 15:00 Kazuyuki Yagasaki (矢ヶ崎 一幸)
Nonintegrability and chaos in Hamiltonian systems with saddle-centers

15:00 – 15:20 Tea Time

15:20 – 16:10 Wolfram Bauer
The fundamental solution of a class of ultra-hyperbolic operators on pseudo-H-type groups

16:20 – 17:10 Tohru Morimoto (森本 徹)
Extrinsic geometries and differential equations

Wednesday, 19/December/2018

09:30 – 10:20 Elmar Schrohe
Elliptic operators associated with groups of quantized canonical transformations

10:20 – 10:40 Tea Time

10:40 – 11:30 Sonja Hohloch
Dynamics around focus-focus singularities in semitoric systems

11:40 – 12:30 Wei Wang
The tangential k-Cauchy-Fueter complexes and Hartogs' phenomenon over the right quaternionic Heisenberg group

12:30 – 14:00 Lunch Time

14:00 – 14:50 Nguyen Tien Zung
On a universal conservation law in dynamics

14:50 – 15:10 Tea Time

15:10 – 16:00 Masanori Adachi (足立 真訓)
Sobolev estimates for the complex Green operator on Levi-flat manifolds

16:10 – 16:50 Le Bich Phuong
Convergence of stochastic gradient flows

15:55 – 16:35 Abdellah Laaroussi
Weyl asymptotics of a class of elliptic and hypoelliptic operators: methods and problems

Thursday, 20/December/2018

09:30 – 10:20 Irina Markina
Extremal curves on Stiefel and Grassmann manifolds

10:20 – 10:40 Tea Time

10:40 – 11:30 CHisato Iwasaki (岩崎 千里)
On the heat kernel for forms on the Heisenberg group

11:40 – 12:30 Dmitry V. Zenkov
Hamel's Formalism for Infinite-Dimensional Mechanical Systems

12:30 – 14:00 Lunch Time

14:00 – 14:50 Uwano Yoshio (上野 嘉夫)
Symmetry and reduction for the α-geodesics on quantum statistical manifolds

14:50 – 15:10 Tea Time

15:10 – 15:50 Sogo Yamanaka (山中 祥五)
Nonintegrability of three-degree-of-freedom Birkhoff normal forms of resonance degree two

15:55 –16:35 Shoya Motonaga (本永 翔也)
Nonpersistence of periodic orbits, homoclinic orbits, first integrals, and commutative vector fields in perturbed systems

18:00 – Banquet at Lake Biwa Prince Hotel Otsu

Friday, 21/December/2018

09:30–10:20 Tudor S. Ratiu
Differential character valued momentum maps

10:30 – 11:20 Toshihiro Iwai (岩井 敏洋)
The 2D Kramers-Dirac oscillator

11:20 – 11:40 Tea Time

11:40 – 12:30 Linyu Peng
Symmetries of semi-discrete variational problems and Noether's theorems

12:40 – 13:30 Hiroaki Yoshimura (吉村 浩明)
A variational formulation, Dirac structures and dynamical systems for nonequilibrium thermodynamics

13:30 – 13:40 Closing

(2) Abstracts

Title: Integrable Systems: The Clebsch top and its associated Kummer Surface

Jean-Pierre Françoise
Sorbonne Université, France

Abstract:
This is a joint article with Daisuke Tarama. It contains a comprehensive presentation of a classical article of Weber (1878) about the Clebsch case of the motion of a rigid body in an ideal fluid. The article is revisited in the light of several more recent contributions of Aomoto (1981), Enolski-Fedorov (2016) and Magri-Skrypyk (2016). We emphasize in particular the existence of an invariant Kummer surface and the algebraic linearization on the Jacobian of an hyperelliptic curve of genus two. We further discuss the existence of invariant subspaces on which the motion is equivalent to the free rigid body. Finally, we introduce an explicit computation of the action-angle coordinates.

Title: Nonintegrability and chaos in Hamiltonian systems with saddle-centers

Kazuyuki Yagasaki
Kyoto University, Japan

Abstract:
Nonintegrability and chaos are fundamental and central topics not only in Hamiltonian systems but also in dynamical systems, as seen in the famous research of Poincare. It is clear that if it exhibits chaos, then a dynamical system is nonintegrable. However, the converse of this statement may not necessarily hold. In this talk, we consider Hamiltonian systems with saddle-centers and discuss this problem. Our main tools are Melnikov-type methods for existence of chaos and the Morales-Ramis theory for nonintegrability. Joint work with Shogo Yamanaka is included.

Title: The fundamental solution of a class of ultra-hyperbolic operators on pseudo-H-type groups

Wolfram Bauer
Leibniz Universität Hannover, Germany

Abstract:
Pseudo-H-type Lie groups $G_{r,s}$ of signature (r, s) are defined via a module action of the Clifford algebra $C\ell_{r,s}$ on a vector space $V \cong \mathbb{R}^{2n}$. They form a subclass of all 2-step nilpotent Lie groups and based on their algebraic structure they can be equipped with a left-invariant pseudo-Riemannian metric. Let $\mathcal{N}_{r,s}$ denote the Lie algebra corresponding to $G_{r,s}$. A choice of left-invariant vector fields $[X_1, \cdots, X_{2n}]$ which generate a complement of the center of $\mathcal{N}_{r,s}$ gives rise to a second order operator

$$\Delta_{r,s} := \left(X_1^2 + \cdots + X_n^2\right) - \left(X_{n+1}^2 + \cdots + X_{2n}^2\right),$$

which we call ultra-hyperbolic. In terms of classical special functions we present families of fundamental solutions of $\Delta_{r,s}$ in the case $r = 0$, $s > 0$ and study their properties. In the case of $r > 0$ the operator $\Delta_{r,s}$ admits no fundamental solution in the space tempered distributions. This is a joint work with I. Markina (University of Bergen) and A. Froehly (formerly Leibniz Universität Hannover).

References

[1] W. Bauer, K. Furutani, and C. Iwasaki, *Spectral zeta function on pseudo H-type nilmanifolds*, Indian J. Pure Appl. Math. **46** (4) (2015), 539-582.

[2] P. Ciatti, *Scalar products on Clifford modules and pseudo-H-type Lie algebras*, Ann. Mat. Pura Appl. **178** (4), 132 (2000).

[3] K. Furutani, I. Markina, *Complete classification of pseudo- H-type Lie algebras: I*, Geom. Dedicata **190** (2017) 23-51.

[4] D. Müller, and F. Ricci, *Analysis of second order differential operators on Heisenberg groups I*, Invent. Math. **101** (1990), 545-582.

Title: Extrinsic geometries and differential equations

Tohru Morimoto
Seki Kowa Institute for Mathematics and Institut Mathématiques de Kiyoshi Oka, Japan

Abstract:

The distinction between intrinsic and extrinsic geometry is fundamental in geometry. The former treats spaces and the latter figures. A space may be defined to be a set S equipped with a geometric structure σ on S. A figure may be defined to be a morphism $\varphi\colon X \to A$ of spaces, where the target space A is usually assumed to be a homogeneous space L/L^0 of a group L factored by a subgroup L^0.
Two spaces X and Y are said to be (intrinsically) equivalent if there exists an isomorphism $f\colon X \to Y$. Two figures $\varphi\colon X \to L/L^0$ and $\psi\colon Y \to L/L^0$ are said to be (extrinsically) equivalent if there exist an isomorphism $f\colon X \to Y$ and $a \in L$ such that $\Lambda_a \circ \varphi = \psi \circ f$, where Λ_a denotes the left translation by a.
Especially in smooth or analytic category one of the central problems is to find the complete invariants of a space X or a figure $\varphi\colon X \to L/L^0$ in order to solve the so called equivalence problems, that is, to decide whether two spaces or two figures are (locally) equivalent or not.
For intrinsic geometry we have general theories originated by Cartan and developed mainly by Kuranishi, Spencer, Singer- Sternberg, Tanaka, and Morimoto (cf [2]).
In this talk I will give a unified scheme to treat the equivalence problem for extrinsic geometry, following [1]. In particular, I will show a general method to construct the complete invariants of a morphem

$$\varphi\colon (M, \mathfrak{f}) \to L/L^0 \subset \mathrm{Flag}(V, \phi)$$

from a filtered manifold $(M, \mathfrak{f})$ to a flag variety $\mathrm{Flag}(V, \phi)$ of type ϕ, in which we prove that the invariants live in the cohomology spac $H^1_+(\mathfrak{g}_-, \mathfrak{l}/\hat{\mathfrak{g}})$ associated to the symbol of φ.
I will also discuss close relations among the geometries intrinsic and extrinsic, and the differential equations linear and non-linear, in particular, the functorial isomorphism between the category of extrinsic geometries in flag varieties and the category of involutive systems of linear differential equations, and various results and problems which arise from these points of view.

References

[1] B. Doubrov, Y. Machida, and T. Morimoto, *Extrinsic geometries and linear differential equations*, in preparation.
[2] T. Morimoto, *Lie algebras, geometric structures and differential equations on filtered manifolds*, Advanced Studies in Pure Mathematics **37**, 2002, 205–252.

Title: Elliptic operators associated with groups of quantized canonical transformations

Elmar Schrohe
Leivniz Universität Hannover, Germany

Abstract:
Given a closed manifold M and a discrete group G of quantized canonical transformations Φ_g, $g \in G$, we are developing an index theory for the algebra of operators of the form

$$A = \sum_g D_g \Phi_g$$

on $L^2(M)$, where the sum is finite and the D_g are zero order pseudodifferential operators. This framework encompasses a variety of situations studied earlier, such as the Atiyah-Weinstein index problem or the index theory for operators with shifts.

As a first step we introduced in [1] suitable notions of symbols and ellipticity and showed the Fredholm property of elliptic elements. Here, also the more general case of operators of the form $I + \int_G D_g \Phi_g \, d\mu_G(g)$ for finite-dimensional Lie groups was considered.

As a first step towards an index formula, we focus on the case of finite extensions of abelian groups. We introduce the localized algebraic index of the complete symbol of an elliptic operator. With the help of a calculus of semiclassical quantized canonical transformations, a version of Egorov's theorem and a theorem on trace asymptotics for semiclassical Fourier integral operators we show that the localized analytic index and the localized algebraic index coincide. As a corollary, we express the Fredholm index in terms of the algebraic index. In forthcoming work we will try to apply methods of algebraic index theory in order to obtain an index formula.

Joint work with Anton Savin and Boris Sternin.

References

[1] A. Savin, E. Schrohe, B. Sternin, Elliptic operators associated with groups of quantized canonical transformations, *Bull. Sci. Math.*, to appear.

Title: Dynamics around focus-focus singularities in semitoric systems

Sonja Hohloch
University of Antwerp, Belgium

Abstract:
A semitoric integrable Hamiltonian system, briefly a semitoric system, is given by two autonomous Hamiltonian systems on a 4-dimensional manifold whose flows Poisson-commute and induce an $(\mathbb{S}^1 \times \mathbb{R})$-action that has only nondegenerate, nonhyperbolic singularities. Semitoric systems have been symplectically classified a few years ago by Pelayo & Vu Ngoc by means of five invariants.

Three of these five invariants are the so-called *Taylor series invariant*, the *height invariant*, and the *twisting index*. Roughly, the first one describes the behaviour of the system near the focus-focus singular fibre, the second one the position of the focus-focus value, and the third one compares the 'rotation' within regular fibres close to one focus-focus singularity to that close to another focus-focus singularity.

Recently there has been made considerable progress in understanding and computing these invariants and, in this talk, we present the (results of the) finished and ongoing projects with J. Alonso (Antwerp), H. Dullin (Sydney), and J. Palmer (Rutgers):

- Taylor series and twisting index for coupled spin oscillators.
- Taylor series, height invariant, and twisting index for coupled angular momenta.
- Putting the Taylor series and twisting index in relation with wellknown notions from classical dynamical systems like rotation number and rotation vector etc.
- Change of the Taylor series and twisting index when varying the parameters of these systems.

References:

- *Symplectic classification of coupled angular momenta* (with J. Alonso and H. Dullin), 59p., arXiv:1808.05849.
- *Taylor series and twisting-index invariants of coupled spin-oscillators* (with J. Alonso and H. Dullin), to appear in Journal of Geometry and Physics.
- *A family of compact semitoric systems with two focus-focus singularities* (with J. Palmer), Journal of Geometric Mechanics 2018, 10(3): 331–357.

Title: The tangential k-Cauchy-Fueter complexes and Hartogs' phenomenon over the right quaternionic Heisenberg group

Wei Wang
Zhejiang University, China

Abstract:
I will discuss the construction of the tangential k-Cauchy-Fueter complexes on the right quaternionic Heisenberg group, as the quaternionic counterpart of $\bar{\partial}_b$ complex on the Heisenberg group in the theory of several complex variables. We can use the L^2 estimate to solve the nonhomogeneous tangential k-Cauchy-Fueter equation under the compatibility condition over this group modulo a lattice. This solution has an important vanishing property when the group is higher dimensional. It allows us to prove the Hartogs' extension phenomenon for k-CF functions, which are the quaternionic counterpart of CR functions.

Title: On a universal conservation law in dynamics

Nguyen Tien Zung
University of Toulouse, France

Abstract:
In this talk I want to explain the following conservation law, its manifestations and applications, especially in the local case near a singular point of a dynamical system: anything which is conserved by a dynamical system is also conserved by its associated torus actions. Part of the talk is based on a joint work with Kai Jiang and Tudor Ratiu on simultaneous normalization of dynamical systems with a singular underlying geometric structure.

Title: Sobolev estimates for the complex Green operator on Levi-flat manifolds

Masanori Adachi
Shizuoka University, Japan

Abstract:
Levi-flat manifolds are odd-dimensional real manifolds that has a foliation of real codimension one whose leaves admit smoothly varying complex structures. Although the leafwise Cauchy-Riemann operator is not hypoelliptic, it enjoys a Sobolev estimate of finite order due to Ohsawa-Sibony (2000). In this talk, I would like to explain this Sobolev estimate in detail for a special but interesting case, the unit circle bundle of hyperbolic Riemann surface endowed with unstable foliation of the geodesic flow.

Title: Convergence of stochastic gradient flows

Le Bich Phuong
Hanoi University of Mining and Geology, Vietnam

Abstract:
In machine learning one often uses a stochastic gradient flow with respect to a loss function on the space of parameters of a neural net to find an optimal prediction model. In this talk, which is based on a work in progress under the supervision of Prof. Nguyen Tien Zung, I want to discuss, via simple toy models, two mathematical problems affecting this general differential learning method, namely:
- In general, the stochastic gradient flow does not converge to the global minimal point of the loss in general (due to the noise), and only to a stochastic equilibrium near a global minimal point. The sharper the loss function, the higher the noise, and the further away the stochastic equilibrium will be from the minimal point.
- Also, in general, the minimal point of the loss function is not the optimal solution for the artificial intelligence problem, due to the bias created by "data imbalance". If the data is too much imbalanced then the minimal point of the loss function is actually a very bad solution.

Title:Weyl asymptotics of a class of elliptic and hypoelliptic operators: methods and problems

Abdellah Laaroussi
Leibniz University Hannover, Germany

Abstract:
We discuss the asymptotics of the eigenvalue counting function of elliptic and hypoelliptic operators. In the elliptic case, there are different methods to obtain such asymptotics, such as method of the hyperbolic operator, asymptotics of the heat trace. In the hypoelliptic case, we focus on the sub-Laplacian on 2-step Nilmanifolds and give some results and open problems.

Title: Extremal curves on Stiefel and Grassmann manifolds

Irina Markina
University of Bergen, Norway

Abstract:
In the talk we describe various left-invariant sub-Riemannian systems on Lie groups that admit explicit solutions with certain properties, and provides geometric origins for a class of important curves on Stiefel manifolds, called quasi-geodesics, that project on Grassmann manifolds as Riemannian geodesics. We show that quasi-geodesics are the projections of sub-Riemannian geodesics generated by certain left-invariant distributions on Lie groups that act transitively on each Stiefel manifold. This is join work with V. Jurdjevic, University of Toronto, Canada, and F. Silva Leite, University of Coimbra, Portugal.

Title: On the heat kernel for forms on the Heisenberg group

Chisato Iwasaki
University of Hyogo, Japan

Abstract:
TBA

Title: Hamel's Formalism for Infinite-Dimensional Mechanical Systems

Dmitry V. Zenkov
North Carolina State University, USA

Abstract:
Separation of the position and velocity measurements in mechanics originated in Euler's work on the rigid body and fluid dynamics. Hamel extended this formalism from the rigid body setting to arbitrary finite-dimensional Lagrangian mechanical systems. This formalism proved to be useful in the mechanics and numerical simulations of multibody systems. This talk will introduce Hamel's formalism for infinite-dimensional mechanical systems, with an emphasis on the dynamics of constrained systems with symmetry.

Title: Symmetry and reduction for the α-geodesics on quantum statistical manifolds

Yoshio Uwano
Kyoto Pharmaceutical University, Japan

Abstract:
This talk aims to report on the 'matrix averaged Hebbian learning equations' (MAHLE) whose trajectories realize the exponential-type (e-) geodesics on the quantum statistical manifolds. The Hamiltonian system describing the MAHLE is shown to be in quadrature. The symplectic reduction technique works well to show the quadrature. This may be the first attempt to develop Hamiltonian mechanical study on the cotangent-bundle QSM [a]. The quantum statistical manifold, denoted by Q_n, is the space of $n \times n$ positive-definite Hermitean matrices with unit trace. On Q_n, the MAHLE takes the form

$$\dot{\rho} = (\rho C + C\rho) - 2\mathrm{Tr}\,(C\rho)\rho \quad (\rho \in Q_n), \tag{0.1}$$

where C is a constant Hermitean matrix [b]. The e-geodesics on Q_n are the α-geodesics with $\alpha = 1$ which are the autoparallel curves with respect to the parallel translation,

$$X_0 \in T_{\rho_0} Q_n \mapsto \frac{1}{2}(\rho_1 L_{\rho_0}(X_0) + L_{\rho_0}(X_0)\rho_1) - \mathrm{Tr}\,(\rho_1 L_{\rho_0}(X_0))\rho_1 \in T_{\rho_1} Q_n, \tag{0.2}$$

where both of the tangent spaces, $T_{\rho_j} Q_n$ $(j = 1, 2)$, are identified with the space of traceless Hermitean matrices and $L_{\rho_0}(X_0)$ stands for the SLD (symmetric logarithmic derivertive) of X_0 determined uniquely by $X_0 = (\rho_0 L_{\rho_0}(X_0) + L_{\rho_0}(X_0)\rho_0)/2$.

Proposition 1 (U)**.** *The e-geodesic $\rho(t)$ subject to $\rho(0) = \rho_0$ and $\dot{\rho}(0) = X_0$ is the trajectory of the MAHLE with $C = L_{\rho_0}(X_0)/2$.*

Consider the tangent bundle $T^*H_n^+$ of H_n, the space of positive-definite Hermitean matrices, which can be identified with $H_n^+ \times H_n$ where H_n denotes the space of Hermitean matrices. On endowing the canonical symplectic form, say Ω, with $T^*H_n^+$, we can complete the symplectic reduction of $(T^*H_n^+, \Omega, K^\#)$ with $K^\#(r, P) = \mathrm{Tr}\,(CrP + PCr)$ to (T^*Q_n, ω, K) by the $\mathbf{R}$-action, $(r, P) \in T^*H_n^+ \mapsto (e^s r, e^{-s} P) \in T^*H_n^+$ $(s \in \mathbf{R})$, where T^*Q_n is the cotangent bundle of Q_n, ω the canonical symplectic form on that.

Proposition 2 (U)**.** *The Hamiltonian system $(T^*H_n^+, \Omega, K^\#)$ is in quadrature.*

Proposition 3 (U)**.** *The Hamiltonian system (T^*Q_n, ω, K) desscribes the MAHLE.*

Referrences

- Y.Nakamura, Japan J. Indust. Appl. Math., **11**, 11-20 (1994).
- Y.Uwano, Mathematical Modelling and Geometry, **4**, 19-33 (2016).
- Y.Uwano, *in preparation*

[a]On concerning classical statistical manifolds (CSM), a pioneering work by Nakamura (1994) is made on the tangent bundle of a CSM.

[b]The condition to C has been made weaker than that in Uwano (2016). Equation (0.1) also describes a variant of Moser's form of the finite nonperiodic Toda lattice extended on Q_n.

Title: Nonintegrability of three-degree-of-freedom Birkhoff normal forms of resonance degree two

Shogo Yamanaka
Kyoto University, Japan

Abstract:
Birkhoff normal forms are special normal forms of Hamiltonian systems near equilibria and the problem of determining whether they are non-integrable has attracted much attention in the field of dynamical systems. We show that three-degree-of-freedom Birkhoff normal forms are non-integrable when they are in $1:2:\omega$ resonance for $\omega = 1, 2$, or 4. Recently, O. Christov [1] also studied non-integrability of the same normal forms although his proofs contained some errors.

References

[1] O. Christov, Non-integrability of first order resonances in Hamiltonian systems in three degrees of freedom, Celest. Mech. Dyn. Astr., 112 (2012), 149-167.

Title: Nonpersistence of periodic orbits, homoclinic orbits, first integrals, and com- mutative vector fields in perturbed systems

Shoya Motonaga
Kyoto University, Japan

Abstract:
Determination of whether periodic orbits, homoclinic orbits, first integrals or commutative vector fields may persist under perturbations is one of the most important problems in the field of dynamical systems. In this talk, we give several theorems on necessary conditions for their persistence in general perturbed systems. Moreover, we consider periodic perturbations of one-degree-of-freedom Hamiltonian systems and describe some relationships between our results and the standard Melnikov method for periodic orbits and homoclinic orbits.

Title :Differential character valued momentum maps

Tudor S. Ratiu
Shanghai Jiao Tong University, China

Abstract:
In this talk I will introduce an extended version of the momentum map with values in differential character groups. This momentum map is able to capture topological invariants that are conserved on the flow of symmetric Hamiltonians. These momentum maps are built on the model of Poisson Lie groups and have important applications when working in infinite dimensions, namely when the group is that of diffeomorphisms preserving certain geometric structures or the automorphism group and the gauge group of a bundle. None of the groups for which this momentum map exists, have a Poisson Lie structure to my knowledge.

The 2D Kramers-Dirac oscillator

Toshihiro Iwai
Kyoto University, Japan

Abstract:
The Dirac oscillator was initially introduced as a Dirac operator which is linear in momentum and position variables, and has been developed with interest in integrable Dirac equations. This talk starts with a review of free Dirac equations with boundary condition and then proceeds to a review of the Dirac oscillators with interest in band rearrangement against a parameter. After that, the 2D Kramers-Dirac oscillator is introduced. In contrast to the usual 2D Dirac oscillator, the 2D Kramers-Dirac oscillator admits the time-reversal symmetry, which is a reason for the present nomenclature. It is shown that there exists an eigenstate associated with an eigenvalue linear in the control parameter, which goes down from positive values to negative values as the parameter varies in the positive direction. The other eigenvalues are broken up into two bands, positive and negative. In conclusion, a relativistic mechanical view of the 2D Kramers-Dirac oscillator is touched upon.

This talk is based on joint works with B. Zhilinskii at Université du Littoral Côte d'Opale.

Title: Symmetries of semi-discrete variational problems and Noether's theorems

Linyu Peng
Waseda University, Japan

Abstract:
Symmetries, local groups of transformations, were introduced by Sophus Lie in the 1880s in order to study solutions of ordinary differential equations. It is evidential that the symmetry method is essential in solving differential equations, e.g. [3]; for instance, almost all solvable differential equations have symmetries behind the mystery. In 1918, exactly one century ago, Emmy Noether published the paper *'Invariante Variationsprobleme'* with the celebrated Noether Theorems [2]. Her first theorem tells that every symmetry of an action, the integral of a Lagrangian function, amounts to a conservation law of the Euler–Lagrange equations, while her second theorem establishes a one-to-one correspondence between infinite-dimensional Lie algebra of variational symmetries and differential relations among the Euler–Lagrange equations.

Noether's theorems have been extended to finite difference and finite element variational problems since 1980s, leaving the semi-discrete case a blank page even though symmetry analysis of semi-discrete equations were greatly investigated for almost three decades, e.g. [1]. The main difficulty may lie on the lack of a proper symmetry prolongation, in particular the lack of an evolutionary representative making the integration and summation by parts accessible [4]. In this talk, I will show how we can possibly overcome such a difficulty and the extension of Noether's theorems to semi-discrete equations.

References

[1] D. Levi and P. Winternitz, Symmetries and conditional symmetries of differential-difference equations, *J. Math. Phys.* **34** (1993), 3713–3730.

[2] E. Noether, Invariante Variationsprobleme, *Nachr. v. d. Ges. d. Wiss. zu Göttingen, Math-phys. Klasse* **2** (1918), 235–257. (English transl.: *Transport Theory Statist. Phys.* **1** (1971), 186–207.)

[3] P. J. Olver, *Applications of Lie Groups to Differential Equations, (2nd ed.)*, Springer-Verlag, New York, 1993.

[4] L. Peng, Symmetries, conservation laws, and Noether's theorem for differential-difference equations, *Stud. Appl. Math.* **139** (2017), 457–502.

Title: A variational formulation, Dirac structures and dynamical systems for nonequilibrium thermodynamics

Hiroaki Yoshimura
Waseda University, Japan

Abstract:
We present a Lagrangian variational formulation for nonequilibrium thermodynamic, which is a natural extension of Hamilton's principle in mechanics to thermodynamics including irreversible processes. The irreversibility can be encoded into a nonlinear phenomenological constraint, which is represented by nonlinear nonholonomic constraints due to the entropy production associated with all the irreversible processes. We show that the variational formulation enables to treat various (finite dimensional) discrete systems such as mechanical systems with friction, matter transfer, electric circuits, chemical reactions, and diffusion across membranes as well as (infinite dimensional) continuum systems such as multicomponent reacting viscous fluids. In addition to the variational formulation, we clarify the underlying structures of the nonequilibrium thermodynamics, where the nonlinear nonholonomic constraint of thermodynamic type can be incorporated into Dirac structures and with the associated Dirac dynamical systems.

(3) Organizers

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